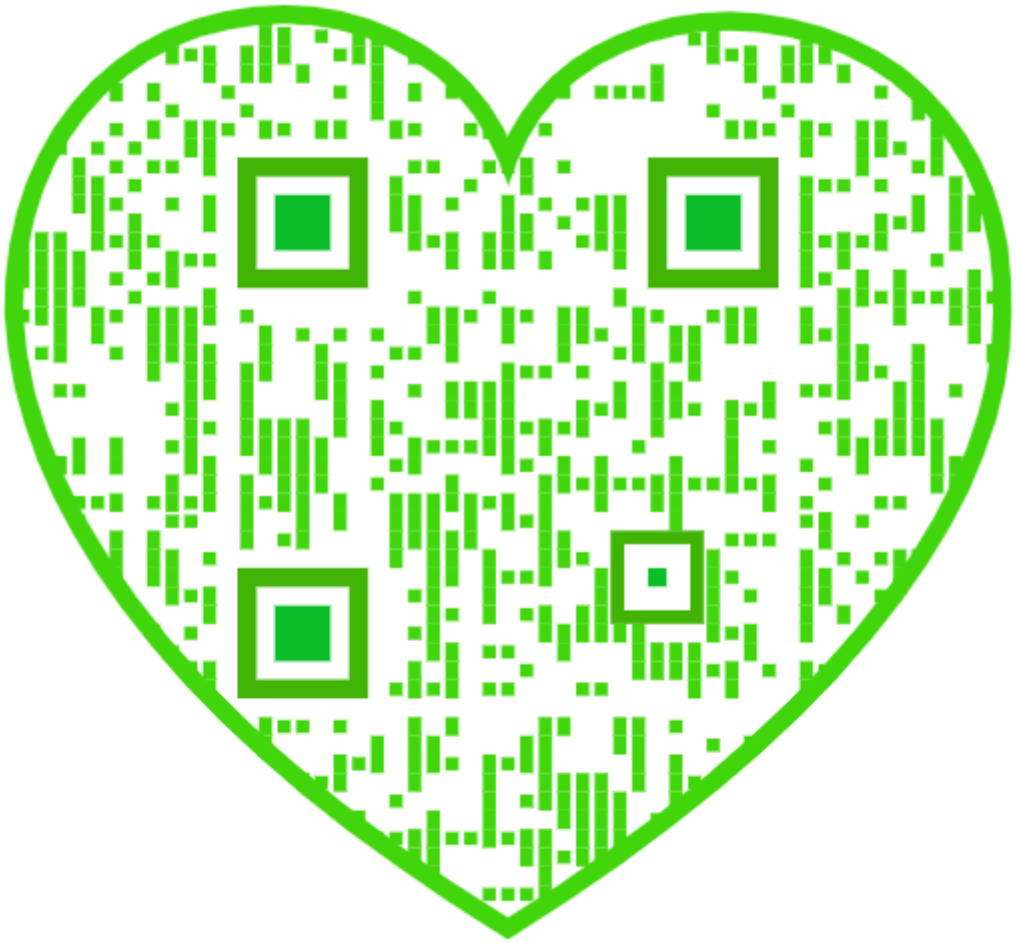

THE

FRANKLIN

ELEMENTARY ARITHMETIC

BY

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BOSTON

WILLIAM WARE AND COMPANY

[SUCCESSORS TO BREWER AND TILESTON]

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PREFACE.

THE *Franklin Elementary Arithmetic*, though designed to be an introduction to the *Franklin Written Arithmetic*, is, nevertheless, a complete arithmetic of its kind. It contains a short course in the elements of numbers, with such applications as are necessary in ordinary business transactions. With the oral teaching which should accompany the use of any book, this book contains enough to meet the wants of that large class of pupils who leave the schools at twelve or thirteen years of age, and of all who desire to master the essentials of arithmetic but have not time to study all that is found in the larger books.

The method of the book is indicated by its title. It distinguishes arithmetical operations from the science of arithmetic. While it gives a systematic practice in the former, it leaves the latter to be learned after the mere operations have become familiar by practice. No attempt is made to establish general principles, but the pupil is led to operate by imitating processes illustrated by simple examples, generally in the concrete. General statements and formal rules are thus rendered unnecessary.

The special features of the work are:—

1. The uniting of oral exercises with the written work, so that the same analysis answers for both processes.

2. The thorough manner of treating the four fundamental operations, especially multiplication and division. These operations are so arranged and combined, that the oral and written processes can be learned in the time ordinarily given to learning the multiplication and division tables.

3. The introduction of United States money and denominate numbers into the same sections with simple numbers and fractions. In this way the reductions in United States money and compound numbers are made a part of the necessary practice in the fundamental operations, and thus much time is saved.

4. Only one method is given for each operation. The pupil is thus spared the perplexity which often results from a multiplicity of methods.

5. The Appendix to the book. This contains full sets of tables, with other methods of operation, to be used instead of those given in the body of the book, if the teacher prefers.

6. Drill Tables, by which the pupil's work may be indefinitely extended, without the teacher being required to search in other books for examples to apply as tests in class exercises.

7. The adaptation of the exercises for use in the class-room, — all oral and illustrative examples being designated by letters of the alphabet, while the examples for the slate and blackboard are numbered consecutively through the section.

BOSTON, August, 1878.

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ELEMENTARY ARITHMETIC.

SECTION I.

READING AND WRITING NUMBERS.

Numbers from One to Ten.

One man.	One.	1.
Two horses.	Two.	2.
Three dogs.	Three.	3.
Four boys.	Four.	4.
Five trees.	Five.	5.
Six cows.	Six.	6.
Seven sheep.	Seven.	7.
Eight birds.	Eight.	8.
Nine posts.	Nine.	9.
Ten rails.	Ten.	10.

ARTICLE 1. How many men do you see in the picture? How many horses? boys? dogs? rails? trees? birds?

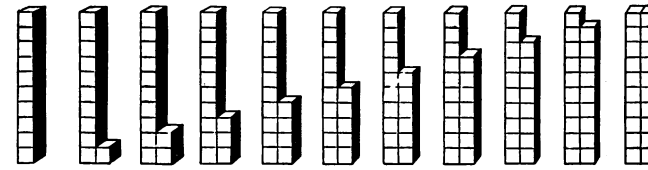
In answering these questions, you have named some numbers.

Name the numbers from one to ten; from ten to one.

2. Copy on your slate the figures that stand for the numbers you have named.

1 2 3 4 5 6 7 8 9 10

Numbers from Ten to Twenty.



10. Ten.....
11. Eleven.....
12. Twelve.....
13. Thirteen...
14. Fourteen..
15. Fifteen....
16. Sixteen....
17. Seventeen.
18. Eighteen..
19. Nineteen..
20. Twenty....

3. How many blocks are

a. Ten blocks and one block? 11 means ten and what?
b. Ten blocks and two blocks? 12 means ten and what?
c. Ten blocks and three blocks? 13 means ten and what?
d. Ten blocks and four blocks? 14 means ten and what?
e. Ten blocks and five blocks? 15 means ten and what?
f. Ten blocks and six blocks? 16 means ten and what?
g. Ten blocks and seven blocks? 17 means ten and what?
h. Ten blocks and eight blocks? 18 means ten and what?
i. Ten blocks and nine blocks? 19 means ten and what?
j. Ten blocks and ten blocks? 20 means how many tens?

Exercises.

4. Count by ones from ten to twenty; from twenty to ten.

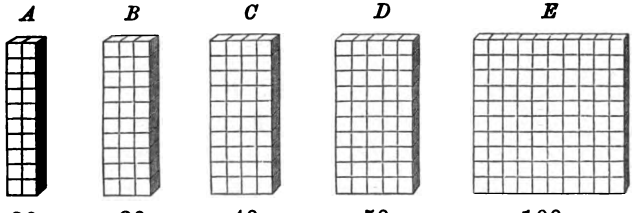
5. Read the following:

k. 11.	*m.* 18.	*o.* 15.	*q.* 19.	*s.* 16.
l. 13.	*n.* 17.	*p.* 12.	*r.* 14.	*t.* 20.

6. Write in figures on your slate all the numbers from ten to twenty.

10 11 12 13 14 15 16 17 18 19 20

Numbers made up of Tens.



20.	30.	40.	50.	100.
Twenty.	Thirty.	Forty.	Fifty.	A hundred.

7. From the collections of blocks represented above find

a. How many tens in twenty?
b. How many tens in thirty?
c. How many tens in forty?
d. How many tens in fifty?
e. How many tens in a hundred?

8. Twenty is written 20. Sixty is written 60.
Thirty is written 30. Seventy is written 70.
Forty is written 40. Eighty is written 80.
Fifty is written 50. Ninety is written 90.
A hundred is written 100.

9. The figure 0 is called ***zero,*** and stands for *no number.*

How many figures are required to write any number of tens to nine tens? Which figure shows how many tens there are?

Exercises.

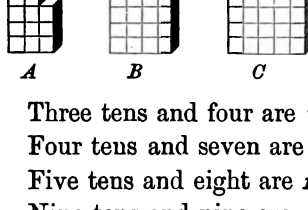
10. Count by tens from ten to one hundred; from one hundred to ten.

11. Read the following:

f. 20.	*h.* 30.	*j.* 90.	*l.* 60.	*n.* 10.
g. 40.	*i.* 70.	*k.* 50.	*m.* 80.	*o.* 100.

12. Write in figures the numbers made up of tens from ten to one hundred.

Numbers made up of Tens and Ones.



A *B* *C*

13. How many tens and how many ones are there in the collection marked A? in B? in C?

Two tens and three are ***twenty-three,*** written 23.

Three tens and four are ***thirty-four,*** written 34.

Four tens and seven are ***forty-seven,*** written 47.

Five tens and eight are ***fifty-eight,*** written 58.

Nine tens and nine are ***ninety-nine,*** written 99.

How many figures are needed to write tens with ones? What does the left-hand figure stand for? What does the right-hand figure stand for?

14. Ones are called ***units,*** and the place at the right where the ones are written is the ***units'*** place. The second place from the right, where the tens are written, is the ***tens'*** place.

Exercises.

15. Count by ones

a. From 20 to 30. *c.* From 30 to 50. *e.* From 50 to 70.

b. From 30 to 20. *d.* From 50 to 30. *f.* From 70 to 100.

16. Read the following:

g. 28.	*j.* 49.	*m.* 50.	*p.* 77.	*s.* 95.
h. 35.	*k.* 31.	*n.* 63.	*q.* 84.	*t.* 46.
i. 42.	*l.* 27.	*o.* 59.	*r.* 69.	*u.* 81.

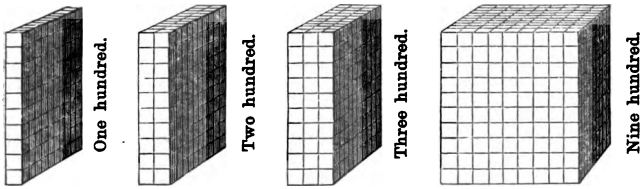
17. Turn to page 32, and read all the numbers written in *A*.

18. Read again the numbers written in *A*, and tell how many tens and how many units there are in each number. Thus, "Fourteen: one ten and four units." "Twenty-six: two tens and six units," and so on.

19. Write in order all the numbers from fifty to ninety-nine.

Numbers made up of Hundreds.

20. Count by hundreds from one hundred to ten hundred.



One hundred is written 100. Three hundred is written 300.

Two hundred is written 200. Nine hundred is written 900.

Exercises.

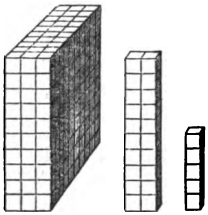
21. Read the following:

a. 400.	*c.* 700.	*e.* 500.	*g.* 900.
b. 300.	*d.* 200.	*f.* 600.	*h.* 800.

How many figures are needed to write hundreds? How many of these figures are zeros?

22. Write in figures the numbers made up of hundreds from one hundred to nine hundred.

Numbers made up of Hundreds, Tens, and Units.



23. Three hundreds, two tens, and five units are three hundred twenty-five, written 325.

In which place from the right are the hundreds written? the tens? the units?

Exercises.

24. Read the following:

i. 123.	*k.* 629.	*m.* 934.	*o.* 206.
j. 304.	*l.* 580.	*n.* 468.	*p.* 370.

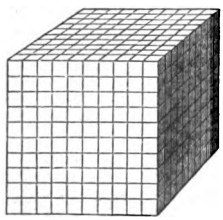
25. Turn to page 32, and read the numbers written in *B*.

26. Write in figures the following:

q. Seven hundred eighty-one.	*t.* Nine hundred fifty.
r. Five hundred fifteen.	*u.* Eight hundred six.
s. Three hundred forty-seven.	*v.* Six hundred eleven.

Let the teacher dictate other numbers made up of hundreds, tens, and units for the pupil to write.

Numbers made up of Thousands.



One thousand.

27. Ten hundreds make a thousand, written 1000 or 1,000.

Count by thousands from one thousand to ten thousand.

Two thousand is written 2000 or 2,000.

Three thousand is written 3000 or 3,000.

And so on.

Exercises.

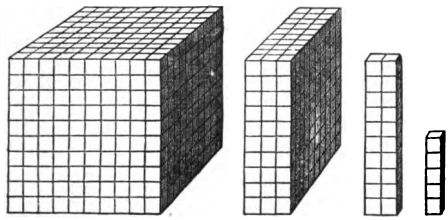
28. Read the following:

a. 1,000.	*d.* 5,000.	*g.* 2,000.
b. 7,000.	*e.* 8,000.	*h.* 4,000.
c. 3,000.	*f.* 6,000.	*i.* 9,000.

How many figures are needed to write thousands? How many of these figures are zeros?

29. Write in figures the numbers made up of thousands from one thousand to nine thousand.

Numbers made up of Thousands, Hundreds, Tens, and Units.



30. One thousand three hundred twenty-five is written 1325.

In which place from the right are the thousands written?

Exercises.

31. Read the following:

j. 1,234.	*m.* 2,304.	*p.* 1,008.
k. 5,067.	*n.* 5,372.	*q.* 3,200.
l. 8,009.	*o.* 8,097.	*r.* 5,970.

32. Turn to page 32, and read the numbers written in *C*.

33. Write in figures:

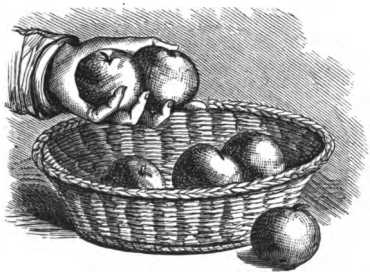
s. One thousand, two hundred.
t. One thousand, two hundred seventeen.
u. Four thousand, six hundred forty-three.
v. Nine thousand, nine hundred eighty.

Let the teacher dictate other numbers made up of thousands, hundreds, tens, and units for the pupil to write.

SECTION II.

ADDITION.

Addition of 1's, 2's, and 3's to other Numbers.



34. How many apples are in the basket?

If the apples in the hand and on the table be put with those in the basket, how many will there be in the basket then?

To find the answer to this question, count the numbers together thus: three, five, six.

35. Counting numbers together is ***adding*** them.

36. The number found by adding is called the ***sum.***

a. Add 4 and 2; 4 and 3; 5 and 2; 5 and 3.

b. What is the sum of 6 and 2? 6 and 3? 7 and 2? 7 and 3?

c. What is the sum of 8 and 2? 8 and 3? 9 and 3? 9 and 2?

Drill Exercises.

37. You will need to do examples like the following over and over again till you can add at sight, that is, till you can name the sum of a pair of numbers as soon as you see the figures.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
d. Add	1	1	1	1	1	1	1	1	1
to	1	3	5	7	4	2	6	8	9
e. Add	2	2	2	2	2	2	2	2	2
to	2	1	4	6	3	5	9	7	8
f. Add	3	3	3	3	3	3	3	3	3
to	2	1	5	3	7	4	8	6	9

38. *g.* How many are 2 and 1? 12 and 1? 22 and 1? 42 and 1?

h. How many are 4 and 2? 14 and 2? 34 and 2? 44 and 2?

i. How many are 9 and 2? 19 and 2? 39 and 2? 59 and 2?

j. How many are 8 and 3? 28 and 3? 48 and 3? 68 and 3?

k. How many are 9 and 3? 19 and 3? 59 and 3? 79 and 3?

39. *l.* Add by 2's from 2 to 20, writing on your slate each sum as you find it; thus,

2, 4, 6, 8, 10, 12, 14, 16, 18, 20.

m. Now look away from your slate and add by 2's without writing the sums.

In the same way,

n. Add by 2's from 1 to 21.

o. Add by 3's from 3 to 30; from 1 to 31; from 2 to 32.

Examples for the Slate.

40. Copy and add the numbers in the following examples.

In Example *a*, begin at the bottom and add thus: "1, 3, 5, 7, 9, 12, 15; sum, 15." Write this sum under the line as it is written in the book. To see if your work is right, begin at the top and add downwards, thus: "3, 6, 8, 10, 12, 14, 15; sum again, 15."

In Example *b*, say "2, 3, 4, 6, 9, 10, 13; sum, 13 tens." Write this sum as it is written in the book, putting a zero to fill the units' place. In the same way do the other examples.

a.	*b.*	(1.)	(2.)	(3.)	(4.)
3	30	2	20	30	300
3	10	3	30	30	100
2	30	2	10	20	300
2	20	1	30	20	200
2	10	3	20	30	100
2	10	2	10	10	300
1	20	2	20	20	300
15	130				

(5.)	(6.)	(7.)	(8.)	(9.)	(10.)
3	2	300	20	300	3
3	2	200	30	300	2
2	1	300	30	300	3
3	1	100	20	200	1
1	2	300	20	200	2
3	3	200	10	100	3
2	3	300	30	300	3
3	2	100	30	200	3
1	3	200	20	200	2

41. In the next examples, beginning with the units, *add the units, the tens, and the hundreds separately.*

(11.)	(12.)	(13.)	(14.)
111	221	102	312
201	101	30	112
322	230	323	310
203	13	112	33
132	323	232	231

42. In the following examples *write the numbers to be added, units under units, tens under tens, and hundreds under hundreds.*

15. Charles counted 201 bricks in one pile, 312 in another, 133 in another, and 123 in another. How many bricks did he count in all?

16. Mrs. Otis has four strawberry-beds. In the first there are 321 plants, in the second 120, in the third 230, and in the fourth 203. How many plants has she in all?

17. Mr. Otis bought a horse for 120 dollars, another horse for 230 dollars, another for 312 dollars, and a carriage for 330 dollars. How many dollars did he pay for all?

18. Add together 123, 301, 22, 210, 300, 123, and 310.

19. Add together 201, 121, 330, 12, 13, 211, and 300.

Addition of 4's and 5's.

43. *a.* If you have 2 cents and earn 4 cents more, how many cents will you have then?

b. 2 and 4 are how many? 3 and 4? 4 and 4? 4 and 3?

c. Charles had 6 cents and Albert gave him 4 cents. How many cents did he then have?

d. How many are 6 and 5? 7 and 5? 7 and 4? 7 and 5?

e. If you should make 8 snowballs and Dwight should make 4, how many would you both make?

f. 8 and 5 are how many? 9 and 5? 9 and 4?

44. Drill Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
g. Add	4	4	4	4	4	4	4	4	4	3	2	3
to	1	3	2	6	8	5	9	7	4	8	7	9
h. Add	5	5	5	5	5	5	5	5	5	2	3	2
to	2	5	9	1	6	3	7	4	8	9	7	6

45. *i.* How many are 4 and 4? 14 and 4? 24 and 4?

j. How many are 7 and 4? 17 and 4? 37 and 4? 47 and 4?

k. How many are 9 and 4? 29 and 4? 49 and 4? 59 and 4?

l. How many are 7 and 5? 27 and 5? 47 and 5? 67 and 5?

m. How many are 9 and 5? 19 and 5? 39 and 5? 79 and 5?

n. How many are 8 and 5? 28 and 5? 58 and 5? 98 and 5?

46. *o.* Add by 4's from 4 to 40, writing down each sum as you find it. Add by 4's from 1 to 41; from 2 to 42; from 3 to 43.

p. Add by 5's from 5 to 50; from 1 to 51; from 2 to 52; from 3 to 53; from 4 to 54.

47. *q.* Name two numbers which added together make 3.

r. Name a pair of numbers which added together make 4; name another pair.

s. Name a pair of numbers which added together make 5; name another pair; name another pair.

Examples for the Slate.

48. Copy and add the following:

(20.)	(21.)	(22.)	(23.)	(24.)	(25.)
2	2	3	30	300	500
3	4	2	40	500	300
3	5	4	20	100	500
5	4	1	50	400	200
3	2	5	30	200	400
4	5	2	40	500	300
1	4	5	50	500	400

49. In adding the units of Example *a* you have 12 units, which you have learned is the same as 1 ten and 2 units (Art. 13), so you may only write down the 2 units, and then add the 1 ten with the tens written in the next column. When adding the tens begin with the 1 ten; thus, "1, 4, 6, 8."

a.
425
324
533
1282

In the same way do the following examples.

(26.)	(27.)	(28.)	(29.)	(30.)
301	405	304	421	330
404	313	225	312	445
423	105	514	204	352
524	432	233	134	35
32	524	22	225	110
414	314	415	542	543

31. Alfred sold 24 tomato-plants to one person, 35 to another, 14 to another, 42 to another, and 35 to another. How many tomato-plants did he sell?

32. John picked up 35 nuts on Monday, 114 Tuesday, 135 Wednesday, 255 Thursday, 54 Friday, and 25 Saturday. How many did he pick up in all?

33. Add together 204 trees, 45 trees, 543 trees, 404 trees, 350 trees, and 55 trees.

Addition of 6's and 7's.

50. *a.* Frank has set out 3 grape-vines, and his father has set out 6. How many grape-vines have both set out?

b. How many are 3 and 6? 4 and 6? 3 and 7? 4 and 7?

51. 4 and 7 may be written 4 + 7. The sign + means *and*. This sign is read "plus."

c. How many are 5 + 6? 5 + 7? 6 + 6? 7 + 6?

d. Mary knit 8 times around her mitten this morning and 6 times this afternoon. How many times did she knit around in all?

e. 7 + 7 equals what number? 8 + 7 equals what?

52. The sign = means "equals," or "is equal to."

f. 7 + 2 + 6 = what number? 6 + 3 + 7 = what?

53. Drill Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
g. Add	6	6	6	6	6	6	6	6	6	4	5	4
to	4	2	1	5	3	9	6	8	7	9	8	7
h. Add	7	7	7	7	7	7	7	7	7	5	4	5
to	5	2	3	1	6	4	8	7	9	9	8	7

54. *i.* 5 + 6 = ? 15 + 6 = ? 25 + 6? 6 + 6? 76 + 6? 86 + 6?

j. 7 + 6 = ? 17 + 6 = ? 37 + 6? 8 + 6? 68 + 6? 78 + 6?

k. 9 + 6 = ? 29 + 6 = ? 49 + 6? 6 + 7? 16 + 7? 46 + 7?

l. 4 + 7 = ? 34 + 7 = ? 54 + 7? 7 + 7? 27 + 7? 47 + 7?

m. 5 + 7 = ? 45 + 7 = ? 75 + 7? 8 + 7? 38 + 7? 88 + 7?

n. 9 + 7 = ? 19 + 7 = ? 69 + 7? 27 + 6? 85 + 7? 48 + 7?

55. *o.* Add by 6's from 6 to 60, writing the sums.

p. Add by 6's from 1 to 61; from 2 to 62; from 3 to 63; from 4 to 64; from 5 to 65.

q. Add by 7's from 7 to 70; from 1 to 71; from 2 to 72; from 3 to 73; from 4 to 74; from 5 to 75; from 6 to 76.

56. *r.* Name each pair of numbers which added together make 6; which make 7; 8; 9; 10.

UNITED STATES MONEY.

57. Ten ***cents*** make a ***dime.***
Ten dimes make a ***dollar.***
One hundred cents make a dollar.

The sign $ stands for dollars. The sign ¢ stands for cents.

58. Examples for the Slate.

34. What is the sum of $37, $502, $657, $462, $16, and $735?

35. What is the sum of 35¢, 72¢, 45¢, 9¢, 102¢, 604¢, and 47¢?

59. Dollars and cents are written together. Thus, twenty-five dollars and thirty-six cents are written $25.36.

(36.)	(37.)	(38.)	(39.)	(40.)
$707	$1.67	$7.12	$12.42	$54.07
674	2.34	4.37	17.32	67.35
547	7.06	3.24	62.45	44.72
436	6.57	2.16	77.23	52.47
776	7.65	5.74	54.36	77.64

60. In order to add the numbers in the following example easily, write them *dollars under dollars and cents under cents.*

41. My sister went shopping, and bought a veil for 72¢, a pair of gloves for $1.25, some slippers for $2.25, some cambric for $3.57, and had $7.65 left. How much money had she at first?

Addition of 8's and 9's.

61. *a.* Ira has 2 school-books and James has 8. How many school-books have both together?

b. How many are 2 + 8? 3 + 8? 3 + 9? 2 + 9?

c. Ira made 4 paper bags to-day and James made 8. How many paper bags did both make?

d. How many are 4 + 8? 4 + 9? 5 + 8? 5 + 9?

e. If you have 6 pencils and buy 8 more, how many pencils will you then have?

f. Add together 7 pencils and 8 pencils; 6 pencils and 9 pencils.

g. 2 + 5 + 9 = what? 2 + 6 + 8? 5 + 3 + 9? 5 + 4 + 9?

62. Drill Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
h. Add	8	8	8	8	8	8	8	8	8	6	7	6
to	6	8	5	2	3	1	4	7	9	7	6	8
i. Add	9	9	9	9	9	9	9	9	9	7	6	7
to	3	6	2	1	5	4	8	7	9	8	9	7

63. *j.* 4 + 8? 14 + 8? 34 + 8? 6 + 8? 26 + 8? 66 + 8?

k. 5 + 8? 25 + 8? 45 + 8? 8 + 8? 38 + 8? 68 + 8?

l. 9 + 8? 39 + 8? 69 + 8? 7 + 8? 17 + 8? 27 + 8?

m. 4 + 9? 44 + 9? 94 + 9? 6 + 9? 16 + 9? 36 + 9?

n. 5 + 9? 25 + 9? 45 + 9? 7 + 9? 27 + 9? 57 + 9?

o. 8 + 9? 18 + 9? 48 + 9? 9 + 9? 29 + 9? 99 + 9?

64. *p.* Add by 8's from 8 to 80; from 1 to 81; from 2 to 82; from 3 to 83; from 4 to 84; from 5 to 85; from 6 to 86; from 7 to 87.

q. Add by 9's from 9 to 90; from 1 to 91; from 2 to 92; from 3 to 93; from 4 to 94; from 5 to 95; from 6 to 96; from 7 to 97; from 8 to 98.

65. *r.* Name each pair of numbers which added together make 11; 12; 13; 14; 15; 16; 17; 18; 19.

66. Examples for the Slate.

Copy and add the following:

(42.)	(43.)	(44.)	(45.)	(46.)
$3.28	$9.19	$0.36	$9.64	$8.19
32.35	8.20	7.21	9.65	7.28
5.92	7.39	8.92	8.70	6.37
8.67	6.48	6.79	2.78	5.00
7.98	5.57	5.43	7.87	6.57
0.83	4.66	2.19	9.86	7.68
5.77	3.75	0.87	6.93	8.79
0.05	2.84	6.54	2.39	9.80

47. I bought a hat for $3.75, a coat for $12, a pair of gloves for $1.75, some handkerchiefs for $3, and a pair of boots for $7.50. What was the cost of all?

48. How much money must you have that you may buy a dictionary for $1.25, a slate for 35¢, a ruler for 12¢, a geography for $1.37, an arithmetic for 92¢, a reader for 87¢, and a writing-desk for $2.35?

49. My father owes $25.78 to one man, $79.48 to another, 97¢ to a third, $75.94 to a fourth, and $5.48 to me. How much money does he owe in all?

50. Charles bought, at a grocery-store, a barrel of flour for $9.35, butter for $17.36, beans for $3.50, fish for 92¢, eggs for 58¢, and $18.15 worth of sugar. What did the whole cost?

51. Mr. Smith spent during the winter months $18.88 for groceries, $15.91 for meat, $20 for fuel, $36 for rent, and $84 for clothing. What did he spend in all?

52. What is the sum of $5.79, $87.68, $9.87, $87.38, $17, and $4.82?

53. What is the sum of $4.35, $25.98, $8.37, 13¢, $38.49, and $1.67?

54. Add $9.98, $17.74, $0.49, $15, $23.05, and 56¢?

For other examples in Addition, see page 33.

SECTION III.

SUBTRACTION.

Subtraction of 1's, 2's, and 3's from other Numbers.

67. What is the whole number of apples in the basket? If the two apples under the hand be taken away, how many will be left in the basket?

68. Taking part of a number away to find how many are left is ***subtracting.***

69. The number, part of which is to be taken away, is the ***minuend.***

70. The part of the minuend to be taken away is the ***subtrahend.***

71. The part of the minuend left after a part has been taken away is the ***remainder.***

a. Subtract 2 from 5; 3 from 5; 2 from 6; 3 from 6.

b. If you subtract 2 from 7, how many will be left?

c. What will be the remainder if 3 be subtracted from 4? from 7? from 8? from 11? from 12?

Drill Exercises.

72. You will need to do examples like the following until you can subtract at sight.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
d. From	2	3	6	5	1	4	8	7	9	10
subtract	1	1	1	1	1	1	1	1	1	1

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
e. From	4	11	5	3	8	6	9	7	2	10
subtract	2	2	2	2	2	2	2	2	2	2
f. From	4	3	7	5	10	12	8	11	6	9
subtract	3	3	3	3	3	3	3	3	3	3

73. *g.* How many are 11 less 2? 21 less 2? 31 less 2?
h. 11 less 3? 21 less 3? 51 less 3? 71 less 3? 91 less 3?
i. 12 less 3? 42 less 3? 62 less 3? 82 less 3? 92 less 3?

74. *j.* Subtract from 20 by 2's, writing down each remainder as you find it; thus, 20, 18, 16, 14, 12, and so on.

k. Now subtract from 20 by 2's without writing the remainders. Subtract in the same way by 2's from 21.

l. Subtract by 3's from 30; from 31; from 32.

Examples for the Slate.

75. Copy the following examples upon your slate and subtract, *taking units from units, tens from tens, hundreds from hundreds,* and writing the remainders as shown in Examples **a** and **b**.

a.	*b.*	(1.)	(2.)	(3.)	(4.)
33	43	56	125	468	359
12	33	20	13	121	202
21	10				

To see if your work is right, *Add the remainder to the subtrahend. The sum ought to equal the minuend.*

76. In Example *c* the 2 tens are taken from 11 tens. Subtract in the same way in the other examples.

c.	(5.)	(6.)	(7.)	(8.)	(9.)
115	114	410	205	911	328
23	21	102	31	103	32
92					

10. If you take 32 from 127, how many will be left?
11. How many are 105 nuts less 22 nuts? 112 less 31?

Subtraction of 4's, 5's, and 6's.

77. *a.* Alvin had 5 melons and sold 4 of them. How many melons had he left?

b. How many are 6 less 4? 7 less 4? 7 less 5? 7 less 6?

78. 7 less 6 may be written $7-6$. The sign $-$ means less. This sign is read "minus" or "less."

c. How many are $8-6$? $8-4$? $8-5$? $9-5$?

d. Jane found 9 ripe peaches and Mary found 4. How many more peaches did Jane find than Mary?

e. How many more are 10 than 4? than 5? than 6?

f. If you have 11 cents and spend 5 of them, how many cents will you have left?

g. If you have 11 and spend 6, how many will you have left?

h. $12-4$ are how many? $12-5$? $12-6$? $13-6$? $13-5$?

79. Drill Exercises.

		(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
i.	From	13	5	8	4	7	11	9	10	6	12
	subtract	4	4	4	4	4	4	4	4	4	4
j.	From	5	14	7	8	11	6	9	13	10	12
	subtract	5	5	5	5	5	5	5	5	5	5
k.	From	10	8	11	14	7	13	12	15	6	9
	subtract	6	6	6	6	6	6	6	6	6	6

80. *l.* How many more are 8 than 4? 18 than 4? 11 than 4? 21 than 4?

m. How many are $12-4$? $22-4$? $13-4$? $53-4$?

n. $11-5$? $21-5$? $41-5$? $12-5$? $32-5$? $42-5$?

o. $13-5$? $23-5$? $53-5$? $14-5$? $44-5$? $64-5$?

p. $11-6$? $21-6$? $31-6$? $13-6$? $23-6$? $43-6$?

81. *q.* Subtract by 4's from 40; from 41; from 42; from 43.

r. Subtract by 5's from 50; from 51; from 52; from 53.

s. Subtract by 6's from 60; from 61; from 62; from 63.

82. Examples for the Slate.

	(12.)	(13.)	(14.)	(15.)	(16.)	(17.)
From	247	125	712	513	138	714
subtract	134	43	405	306	54	206

83. In Example *a*, as we cannot take 4 from 2, we change one of the 8 tens (leaving 7 tens) to units. This 1 ten equals 10 units. Ten units and 2 units are 12 units. We now take 4 units away from 12 units and have 8 units left.

a.

(7) (12)
8 2
2 4
5 8

Next we take 2 tens from 7 tens, and have 5 tens left, and we have for the whole remainder 5 tens and 8 units, or 58.

In subtracting, say "4 from 12, 8; 2 from 7, 5. *Ans.* 58."

In the same way you may do the following examples.

(18.)	(19.)	(20.)	(21.)	(22.)	(23.)
262	374	560	305	618	736
125	236	414	154	432	144

(24.)	(25.)	(26.)	(27.)	(28.)
$21.40	$33.62	$42.34	$80.91	$73.85
15.36	26.16	14.05	25.44	65.16

29. A train of cars took 425 passengers into Buffalo and brought back only 216. How many more passengers were taken into Buffalo than were brought back?

30. In the Bates School are 527 pupils and in the Lincoln School are 455 pupils. How many more pupils are in the Bates School than in the Lincoln?

31. In a certain town are 642 children, of whom only 526 attend school. How many do not attend school?

32. Mrs. Ray bought a house for $2450, but paid only $1625 down. How much does she still owe?

33. James's father made 625 pounds of maple sugar last year, and has made 1242 pounds this year. How many more pounds has he made this year than he made last?

Subtraction of 7's, 8's, and 9's.

84. *a.* Ned's father had 10 horses and sold 7 of them. How many horses had he left?

b. Seven from 10 leaves what? 8 from 10? 9 from 10?

c. Mary is 11 years old and Ethel is 6. What is the difference of their ages?

d. How many are 11 – 8? 11 – 9? 12 – 9? 12 – 7? 12 – 8?

e. John lives 13 miles from his cousin. After going 7 miles on the way, how many more miles has he to go?

f. What number added to 7 will make 13? 14? 15?

g. What number added to 8 will make 13? 14? 15?

h. Emma had 16 rabbits and gave away all but 7. How many did she give away?

i. What number taken from 16 will leave 7? 8? 9?

85. Drill Exercises.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)
j. From	12	11	7	9	8	10	13	15	16	14
subtract	7	7	7	7	7	7	7	7	7	7
k. From	11	13	10	8	12	9	15	14	17	16
subtract	8	8	8	8	8	8	8	8	8	8
l. From	12	17	15	10	11	9	13	16	14	18
subtract	9	9	9	9	9	9	9	9	9	9

86. *m.* 11 – 7? 21 – 7? 41 – 7? 12 – 7? 62 – 7? 72 – 7?

n. 14 – 7? 24 – 7? 54 – 7? 16 – 7? 46 – 7? 96 – 7?

o. 13 – 7? 43 – 7? 83 – 7? 15 – 7? 25 – 7? 75 – 7?

p. 11 – 8? 31 – 8? 51 – 8? 13 – 8? 23 – 8? 53 – 8?

q. 15 – 8? 25 – 8? 45 – 8? 17 – 8? 37 – 8? 87 – 8?

r. 14 – 8? 44 – 8? 84 – 8? 13 – 8? 33 – 8? 73 – 8?

s. 13 – 9? 33 – 9? 63 – 9? 15 – 9? 25 – 9? 85 – 9?

t. 17 – 9? 67 – 9? 87 – 9? 16 – 9? 46 – 9? 96 – 9?

87. *u.* Subtract by 7's from 70; from 71; from 72; from 73; from 74; from 75; from 76.

v. Subtract by 8's from 80; from 81; from 82; from 83; from 84; from 85; from 86; from 87.

w. Subtract by 9's from 90; from 91; from 92; from 93; from 94; from 95; from 96; from 97.

Examples for the Slate.

88. *a.* Subtract 168 from 300.

WRITTEN WORK.

(2) (9) (10)
3 0 0
1 6 8
1 3 2

In this example, as we have no units to take the 8 units from, and no tens to take the 6 tens from, we change one of the hundreds (leaving 2 hundreds) to 10 tens, and then change one of the tens (leaving 9 tens) to ten units. We have now 2 hundreds, 9 tens, and 10 units, from which we can easily take the 1 hundred, 6 tens, and 8 units.

In subtracting, say "8 from 10, 2; 6 from 9, 3; 1 from 2, 1."

(34.)	(35.)	(36.)	(37.)	(38.)	(39.)
400	502	800	$17.28	$98.76	$86.42
285	259	769	5.79	67.89	35.69

40. Mary took $5.10 to the store, and bought a pair of boots for $2.38. How much money had she left?

41. A man bought a horse for $378 and sold him for $417. How many dollars did he gain?

42. Sarah's father bought her a piano for $425, paying all but $287 of the money at the time he bought it. How much did he pay?

43. A house and lot together were worth $4500, and the land alone was worth $1185. What was the house worth?

44. Otis bought of Mr. Carter cloth to the amount of $29.80, and Mr. Carter took off $2.98 of the cost, for present payment. How much did Otis then have to pay?

45. Mr. Diller raised 1280 baskets of peaches this year and 927 baskets last year. How many more baskets of peaches did he raise this year than last?

For other examples in Subtraction, see page 33.

89. Miscellaneous Oral Examples.

a. James has 24 doves, and Walter has 9 more than James. How many doves has Walter?

b. If Burt has 26 chestnuts and gives 9 of them to Ida and 7 to you, how many will he have left?

c. Annie had 50¢, and bought with it some paper for 9¢, some ink for 10¢, and an orange for 6¢. How many cents had she left?

d. Add 9 to 32 and then subtract 7 from the sum.

e. From 59 take the sum of 14 and 6.

f. A boy threw a ball 7 feet beyond the fence. If the fence was 24 feet from the boy, how far must he go to get the ball? How far to get the ball and return?

g. If John is 7 feet from you on the right and Alice 14 feet from you on the left, how far is John from Alice? How far must John walk to go past you to Alice and then back to you again?

h. The end of the school-room is 30 feet long. There are 2 doors in the end each 4 feet wide, and the rest of the distance is filled with blackboards. How many feet are filled with blackboards?

i. Another school-room is 33 feet long. Ellen is standing at one end, and Jane is standing opposite her at the other end. If Jane now walks 5 feet towards Ellen, and Ellen walks 7 feet towards Jane, how far apart will they be?

90. Miscellaneous Examples for the Slate.

46. If a boy earns $9.50 and spends $2.35 for books, how much money will he have left?

47. Grace has $29.63 and Charlotte has $7.85 more than Grace. How much money has Charlotte?

48. Mr. Rolfe owned 435 acres of land and sold 42 acres to one person and 37 acres to another. How many acres had he left?

49. The material for Kate's dress cost $5.84, the making cost $3.15 and the trimming $1.75. What did the whole dress cost?

50. A bookcase has 5 shelves. On the first are 24 books; on the second, 37; on the third, 45; on the fourth, 48; and on the fifth, 52. If 4 books be taken from the first shelf, 8 from the second, and 17 from the third, how many books will remain in the case?

51. Mrs. Bush lent a neighbor $20. He paid her at one time $6.50, at another time $5.85, and at another time gave towards payment of the debt, a calf worth $7.50. How much of the debt did he then owe?

52. Mr. Mason paid $121.80 for his passage to Liverpool, and $26.32 more for his passage back than for his passage out. How much did he pay for his passage back?

53. A grocer paid $18 for a barrel of sugar and $0.75 for freight upon it. For how much must he sell it to gain $6.75?

54. Mr. Gove's income this year is $568 besides his salary of $1500. He pays $250 for rent, $67 for fuel, and a tax of $48. How much has he left for his other expenses?

55. Dora's mother gave her a 5-dollar bill, with which she bought some cambric for $1.55, some thread for 10¢, buttons for 37¢, and a sack pattern for 35¢. How much change should she receive in return?

56. Find the sum of $3684 and $2748, and then subtract from this sum $197.

57. From $30.07 take the sum of $5.87 and 62¢.

58. A man who has $8600 owes $2107 to one person and $3648 to another. What will he have left after paying what he owes?

59. A man owned a house that cost $5285 and a barn that cost $670; he was obliged to sell them both for $4500. How much money did he lose by the sale?

60. John has 264¢, which is 187¢ less than Charles has. How many cents has Charles?

61. Find the difference between 284 and 109, and then take that difference from 248 + 2964 + 307.

62. A man bought a carriage for $465, spent $38 for repairing it, and then sold it for $485. Did he gain or lose, and how much?

63. James shot an arrow 74 feet and Fred shot one 28 feet farther than James. How many feet must Fred go to pick up his arrow and return?

64. Ralph lives 150 yards north of the school-house. One morning he went for Arthur, who lives 85 yards south of the school-house, and then went to school. How many yards did he travel?

65. Two girls start from the same place to roll hoops. One goes east 298 feet, and the other west 284 feet. How far apart are they then?

66. If the two girls are 582 feet apart and should go towards each other, one 132 feet and the other 175 feet, how far apart would they be then?

67. From a tank containing 412 gallons of water 148 gallons were drawn off, after which 327 gallons ran in. How many gallons were there in the tank then?

68. Here is a boy's account of what he received and what he spent last June. How much has he left to begin his July account with?

		$	¢
June 1	*On hand from last month*	*2*	*42*
"	*Received from father*	*5*	*00*
" *4*	*For doing an errand*	.....	*10*
" *17*	*For weeding for Mrs. Bates, 4 hours*	.....	*50*
" *23*	*Mother gave me*	.....	*25*
	Amount		
June 3	*Spent for cap, 75¢; for slate, 37¢*		
" *6*	*Gave Mrs. Fox 40¢; bought ball, 35¢*	.....	
" *8*	*Paid for square of glass, 40¢; for paper, 5¢*	.....	
" *15*	*Birthday's present for Susie*	.....	*25*
" *25*	*Bought shoes, $2.25; hoe, 50¢*		
	Amount		

Balance on hand June 30, $

92. DRILL TABLE No. 1.

Simple Numbers.

Examples.	A *g h*	B *i j k*	C *l mn o*	D *p q*	E *r s t*	F *u vw x*	G *y z*
1.	14	752	4 505	25	324	1 231	23
2.	26	720	5 604	48	747	7 038	38
3.	46	237	6 162	37	421	5 370	49
4.	32	471	7 390	16	908	6 325	37
5.	26	623	8 919	84	162	2 591	72
6.	45	305	7 052	98	143	4 028	15
7.	21	750	7 644	32	581	3 237	81
8.	33	626	5 381	18	645	9 804	66
9.	42	538	2 486	47	970	8 275	75
10.	45	242	1 827	54	352	5 129	54
11.	31	327	9 413	81	147	2 624	29
12.	34	428	3 796	72	638	4 140	91
13.	15	514	4 468	45	902	1 561	34
14.	23	762	8 574	32	540	7 018	87
15.	35	644	7 631	29	412	5 381	55
16.	44	371	3 628	17	915	4 973	43
17.	41	427	2 376	41	109	9 462	76
18.	36	755	6 705	95	484	4 092	21
19.	45	237	8 892	82	723	8 407	83
20.	24	642	4 534	34	528	1 295	17
21.	32	768	2 347	56	892	3 011	45
22.	45	374	3 926	19	717	2 814	62
23.	42	461	9 319	78	158	5 390	36
24.	26	175	9 235	39	243	4 321	96
25.	15	780	1 527	46	905	6 804	58

93. Explanation of the Use of the Drill Tables.

The following illustration will show how the table on page 32 may be used for class drill so as to give each pupil a different example:

1. Let the members of the class number themselves 1, 2, 3, etc., to any given number up to 25; and let each member find his number in the left-hand margin of the table.

2. The teacher then gives a direction in this form: "Add A, D, and G."

3. In obedience to this direction, each pupil will add the numbers that he finds expressed under the letters A, D, and G, and in the line of his own number. Thus, pupil No. 1 will add 14, 25, and 23; No. 2 will add 26, 48, and 38; and so on.

Thus a series of examples is given out at a single dictation, and the pupils are taught to work independently.

The key contains answers to all these examples.

94. Exercises on Table No. 1.

Addition.

1. Add A, D, and G.
2. Add A, B, and C.
3. Add D, E, F, and G.
4. Add B, C, D, and E.
5. Add A, B, C, D, E, and 269.

Subtraction.

6. From B take A.
7. From E take D.
8. From C take B.
9. Subtract E from F.
10. Find the difference between C and F.

Oral Practice.

11. Add g and h; add p and q; add y and z.
12. Add i, j, and k; m, n, and o; r, s, and t.
13. Add l, m, n, o; add u, v, w, x.
14. Add g, h, i, j, k, l, m, n, o; add p, q, r, s, t, u, v, w, x, y, z.
15. In each column g, h, i, etc., add all the numbers.

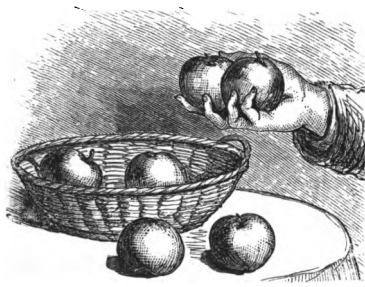
How many are

16. $31-g-h$?
17. $57-g-h$, etc., to l?
18. $85-g-h$, etc., to q?
19. $74-r-s-t$, etc., to z?
20. $100-g-h-i$, etc., to z?

SECTION IV.

MULTIPLICATION.

Multiplication of 2's, 3's, and 4's.



95. How many apples are there in the hand? in the basket? on the table? How many apples in all?

a. How many are three 2's, or 3 times 2?

b. How many are four 2's? five 2's? 6 times 2?

In finding the answer to each of these questions, you united two or more equal numbers.

96. Uniting two or more equal numbers is ***multiplying.***

97. One of the equal numbers to be united is the ***multiplicand.***

98. The number that tells how many equal numbers are to be united is the ***multiplier.***

99. The result obtained by multiplication is the ***product.***

c. In the example "seven 2's are 14," which is the multiplicand? the multiplier? the product?

d. What is the product of 2 multiplied by 8, or 8 times 2?

100. 2 multiplied by 8 may be written 2×8. The sign $\times$ means "multiplied by."

e. What is the product of 2×9? of 2×10?

f. Add by 2's from 2 to 24. The numbers you have named are all the products of 2 up to 24.

Drill Exercises.

101. *g.* Write the products found by multiplying 2's from one 2 to twelve 2's, in the form given below.

h. Repeat from memory what you have written.

i. Repeat the table several times, forward and back.

TABLE.*

One 2 is 2.
Two 2's are 4.
Three 2's are 6.
Etc.

Name the products of the pairs of numbers written below till you can give them in any order at sight.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
j. Multiply	2	2	2	2	2	2	2	2	2	2	2
by	2	4	3	7	5	9	6	11	8	12	10

k. Multiply these numbers again, and add 1 to each product.

In this exercise, think what the products are, but name only the final results. Thus, in multiplying the first, think 4 and then say 5; in multiplying the second, think 8 and then say 9; and so on.

102. *l.* Add by 3's from 3 to 36.

m. Write the table of 3's from one 3 to twelve 3's.

n. Repeat the table from memory forward and back.

TABLE.

One 3 is 3.
Two 3's are 6.
Etc.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
o. Multiply	3	3	3	3	3	3	3	3	3	3	3
by	2	5	10	3	8	4	11	6	12	7	9

p. Multiply these numbers again and add 1 to each product.

q. Multiply again and add 2 to each product.

103. *r.* Add by 4's from 4 to 48.

s. Write the table of 4's from one 4 to twelve 4's.

t. Repeat the table forward and back.

TABLE.

One 4 is 4.
Two 4's are 8.
Etc.

* See Appendix, page 138.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
u. Multiply	4	4	4	4	4	4	4	4	4	4	4
by	3	5	2	7	9	4	10	12	6	8	11

v. Multiply these numbers again and add 1 to each product.

w. Multiply and add 2. *x.* Multiply and add 3.

104. Oral Examples.

a. If a chair has 2 arms, how many arms have 3 chairs? 4 chairs?

Solution. — If a chair has 2 arms, 3 chairs will have 3 times 2 arms, which is 6 arms. *Ans.* 6 arms. (See Appendix, page 139.)

b. If a knife has 3 blades, how many blades have 2 knives? 3 knives?

c. If each of Harry's 6 playmates gave him 3 peaches, how many peaches did he receive?

d. A fork has 4 tines. How many tines have 2 forks? 3 forks? 7 forks?

e. At $4 a week, how many dollars will a person earn in 4 weeks? in 5 weeks? in 10 weeks?

f. What cost 6 mats at $1 each? at $2? at $3? at $4?

g. One dollar equals 2 half dollars. How many half dollars do 2 dollars equal? 3 dollars? 5 dollars?

h. A square has 4 sides. How many sides have 3 squares?

i. A triangle has 3 angles. How many angles have 2 triangles? 3 triangles? 8 triangles?

j. If a pear is cut into 3 equal parts, each of the parts is 1 third. How many thirds in 4 pears? in 5 pears?

105. *k.* How many are three 2's? two 3's? Is there any difference in the answers?

l. How many are 2×4? 4×2? four 3's? three 4's?

m. How many are 3×2? 3 tens $\times$ 2? 3 hundreds $\times$ 2?

n. How many are 3×5? 3 tens $\times$ 5? 3 hundreds $\times$ 5?

o. How many are 4×7? 4 tens $\times$ 7? 4 hundreds $\times$ 7?

p. How many are 4×9? 4 tens $\times$ 9? 4 hundreds $\times$ 9?

Examples for the Slate.

106. Copy the following examples on your slate. Beginning with the units, *multiply the units, the tens, and the hundreds separately,* writing the products as they are written in Examples **a** and **b**.

a.	*b.*
34	432
2	3
68	1296

	(1.)	(2.)	(3.)	(4.)	(5.)
Multiply	313	423	304	402	404
by	3	3	4	5	6

	c.	*d.*
Multiply	43	$2.40
by	6	7
	258	$16.80

In Example *c*, the product of the units is 18. 18 units are equal to 1 ten and 8 units. We write 8 under the line and keep the 1 ten to add to the product of the tens. The entire product is 258. In Example *d*, as the multiplicand is dollars and cents, so the product is dollars and cents.

For the sake of rapid working, use as few words as possible. Thus, in Example *c*, say "*eighteen*, twenty-four, *twenty-five.*" While saying "eighteen," write 8; and while saying "twenty-five," write 5 and 2.

	(6.)	(7.)	(8.)	(9.)	(10.)	(11.)
Multiply	$413	$423	$1.42	$3.14	$3.24	$2.30
by	7	10	11	9	8	12

12. Mr. Granger has 115 sheep and Mr. Oaks has 3 times as many. How many sheep has Mr. Oaks?

13. After travelling 134 miles on a journey, Mr. Niles had 5 times that distance to go. How many miles had he to go?

14. A man bought a boat for $132 and a horse for 4 times that sum. What did he pay for his horse?

15. What will 8 hats cost at $2.40 apiece?

16. What will 9 caps cost at $1.34 apiece?

17. What will 12 chairs cost at $4.42 apiece?

Multiplication of 5's and 6's.

Drill Exercises.

107. *a.* Add by 5's from 5 to 60.

b. Write the table of 5's from one 5 to twelve 5's in the same way that you wrote the tables of 2's, 3's, and 4's.

c. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
d. Multiply	5	5	5	5	5	5	5	5	5	5	5
by	3	8	7	2	9	4	10	12	6	11	5

e. Multiply these numbers again and add 1 to each product.

f. Multiply and add 2. *g.* Multiply and add 3.

h. Multiply and add 4.

108. *i.* Add by 6's from 6 to 72.

j. Write the table of 6's from one 6 to twelve 6's.

k. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
l. Multiply	6	6	6	6	6	6	6	6	6	6	6
by	2	7	10	3	8	11	4	9	12	5	6

m. Multiply these numbers again and add 1 to each product.

n. Multiply and add 2. *p.* Multiply and add 4.

o. Multiply and add 3. *q.* Multiply and add 5.

109. Oral Examples.

r. If there are 5 books on the table and 10 times as many on the shelf, how many are there on the shelf?

s. There are 6 working days in one week. How many are there in 2 weeks? in 5 weeks? in 10 weeks?

t. A cube has 6 faces. How many faces have 3 cubes? 6 cubes? 12 cubes?

u. If five men can do a piece of work in 2 days, in what time can 1 man do it?

110. Examples for the Slate.

	(18.)	(19.)	(20.)	(21.)	(22.)
Multiply	456	564	645	450	536
by	2	3	4	5	6

(23.)	(24.)	(25.)	(26.)	(27.)	(28.)
524	650	$5.46	$6.34	$4.65	$5.42
7	8	9	10	11	12

29. There are 365 days in a common year. How many days are there in 3 common years?

30. There are 366 days in a leap year. How many days are there in 5 common years and 1 leap year?

31. At $5.65 a week for board, what is the cost of 7 weeks' board?

32. At $6.35 a day, what are Mr. Dole's wages for 6 days?

33. Charles has $6.40 and his brother has 8 times as much. How much money has his brother?

34. Mary bought 12 yards of silk at $1.65 a yard. What did it cost her?

35. What must be paid for 11 acres of land at $2.56 an acre?

36. By working 9 hours a day, a man can do a certain piece of work in 156 days. How many days will it take him if he works 1 hour a day?

37. If 6 men can build a fence in 46 days, in how many days can 1 man build it?

38. How many are 10 times 650 trees? 12 times $536?

39. Multiply 456 by 3 and by 4 and add the products.

40. Multiply 564 by 5 and by 6 and add the products.

41. Multiply 654 by 7 and by 8 and add the products.

42. Multiply 346 by 9 and by 11 and add the products.

43. Multiply 666 by 4 and by 6 and add the products.

Multiplication of 7's and 8's.

Drill Exercises.

111. *a.* Add by 7's from 7 to 84.

b. Write the table of 7's from one 7 to twelve 7's.

c. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
d. Multiply	7	7	7	7	7	7	7	7	7	7	7
by	3	6	4	2	7	5	11	9	12	10	8

e. Multiply these numbers again and add 2 to each product.

f. Multiply and add 3. *h.* Multiply and add 5.

g. Multiply and add 4. *i.* Multiply and add 6.

112. *j.* Add by 8's from 8 to 96.

k. Write the table of 8's from one 8 to twelve 8's.

l. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
m. Multiply	8	8	8	8	8	8	8	8	8	8	8
by	7	9	2	8	3	10	4	11	5	12	6

n. Multiply these numbers again and add 3 to each product.

o. Multiply and add 4. *q.* Multiply and add 6.

p. Multiply and add 5. *r.* Multiply and add 7.

113. Oral Examples.

s. At 7 cents a yard, what will 4 yards of ribbon cost?

t. There are 7 days in a week. How many days are there in 2 weeks? in 3 weeks? in 5 weeks? in 9 weeks?

u. There are 8 quarts in 1 peck. How many quarts are there in 2 pecks? in 3 pecks? in 4 pecks?

v. John is 8 years old, and his father is five times as old. How old is his father?

w. Annie has made a quilt with 8 rows of squares in it, 8 squares in each row. How many squares are there in the quilt?

x. If 8 quires of paper can be bought for $1, how much can be bought for $6? for $7?

y. If a piece of work can be done in 2 days by working 8 hours a day, in how many days can it be done by working 1 hour a day?

z. What is the product of 7×10? 7×11? 8×7? 7×8?

114. Examples for the Slate.

	(44.)	(45.)	(46.)	(47.)	(48.)
Multiply	678	875	748	837	728
by	2	3	4	5	6

(49.)	(50.)	(51.)	(52.)	(53.)	(54.)
187	780	878	787	684	768
7	8	9	10	11	12

55. At $2 each, what will 187 pear-trees cost?

At $2 each, 187 pear-trees will cost 187 times $2. But 187 times $2 is the same as 2 times $187. (Art. 105.) Hence we multiply 187 by 2. *Ans.* $374.

56. At $3 a day, what will a person receive for 278 days' work?

57. At 6 cents a pound, what will 168 pounds of sweet potatoes cost?

58. At 11 cents a pound, what will 287 pounds of sugar cost?

59. What will 2 boxes of soap cost, each box containing 74 pounds, at 9 cents a pound?

60. How many days are there in 48 weeks and 1 day?

61. At 8 cents a yard for muslin, what is the cost of 4 pieces, 47 yards in a piece?

62. Multiply 278 by 6, and multiply the product by 5.

63. Multiply 426 by 3, and multiply the product by 7.

64. Multiply 177 by 4, and multiply the product by 9.

65. Multiply 387 by 2, and multiply the product by 11.

Multiplication of 9's and 10's.

Drill Exercises.

115. *a.* Add by 9's from 9 to 108.

b. Write the table of 9's from one 9 to twelve 9's.

c. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
d. Multiply	9	9	9	9	9	9	9	9	9	9	9
by	11	3	9	2	8	12	4	7	10	5	6

e. Multiply these numbers and add 4 to each product.

f. Multiply and add 5.

g. Multiply and add 6.

h. Multiply and add 7.

i. Multiply and add 8.

116. *j.* Write the table of 10's from one 10 to twelve 10's.

k. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
l. Multiply	10	10	10	10	10	10	10	10	10	10	10
by	3	10	12	2	11	9	4	7	5	8	6

m. Multiply these numbers and add 5 to each product.

n. Multiply and add 6.

o. Multiply and add 7.

p. Multiply and add 8.

q. Multiply and add 9.

117. Write upon your slate the numbers from 1 to 9, as they are written below. (See Appendix, p. 139.)

4 2 3 7 9 1 6 5 8

Beginning at the right,

r. Multiply each number by 2 and add 1 to the product.

s. Multiply by 3 and add 2.

t. Multiply by 4 and add 3.

u. Multiply by 5 and add 4.

v. Multiply by 6 and add 5.

w. Multiply by 7 and add 6.

x. Multiply by 8 and add 7.

y. Multiply by 9 and add 8.

z. Multiply by 11 and add 9.

118. Oral Examples.

a. If a man works 9 hours a day, how many hours does he work in 2 days? in 3 days? in 4 days?

b. Mary pays 10 cents a yard for braid. How much does she pay for 4 yards? for 5 yards?

c. If you sleep 9 hours a day, how many hours do you sleep in 6 days? in a week?

d. At 10 cents apiece, what will 6 tops cost? What will 7 tops cost? 8? 11? 12?

e. How many dimes are there in $1? in $6?

f. In one eagle there are $10. How many dimes are there?

g. What is the product of 9×8? 9×10? 9×12?

h. What is the product of 10×11? 10×12?

i. Is there any difference in the number of trees in two orchards, if one has 9 rows of 10 trees each, and the other 10 rows of 9 trees each? Why?

j. How many tenths in one? in two? in ten?

119. Examples for the Slate.

66. Multiply 798 by 3; by 4; and add the products.

67. Multiply 897 by 5; by 6; and add the products.

68. Multiply 590 by 7; by 8; and add the products.

69. Multiply 489 by 9; by 10; and add the products.

70. Multiply 397 by 11; by 12; and add the products.

71. Multiply 1399 by 2; by 3; and add the products.

72. Find the amount which must be paid for 48 cords of wood at $6 a cord, and 96 cords at $5 a cord?

73. What must be paid for 2 barrels of sugar, the first containing 249 pounds at 8 cents a pound, the other 268 pounds at 11 cents a pound?

74. What cost 498 tons of iron at $9 a ton, and 904 tons at $12 a ton?

For other examples in Multiplication by numbers no greater than 12, see page 71.

Multiplication of 11's and 12's.

Drill Exercises.

120. *a.* Add by 11's from 11 to 132.

b. Write the table of 11's from one 11 to twelve 11's.

c. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
d. Multiply	11	11	11	11	11	11	11	11	11	11	11
by	7	4	8	3	9	2	10	5	11	6	12

e. Multiply these numbers and add 5 to each product.

f. Multiply and add 6. *h.* Multiply and add 8.

g. Multiply and add 7. *i.* Multiply and add 9.

121. *j.* Add by 12's from 12 to 144.

k. Write the table of 12's from one 12 to twelve 12's.

l. Repeat the table forward and back.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
m. Multiply	12	12	12	12	12	12	12	12	12	12	12
by	7	10	2	11	3	12	4	9	5	8	6

n. Multiply these numbers and add 5 to each product.

o. Multiply and add 6. *q.* Multiply and add 8.

p. Multiply and add 7. *r.* Multiply and add 9.

122. Oral Examples.

s. If it takes 11 yards to make a dress, how many yards will it take to make 2 dresses? 3 dresses?

t. There are 4 cousins each 11 years old, what is the sum of their ages? They have a teacher 5 times as old as either of them. How old is the teacher?

u. In one foot there are 12 inches. How many inches are there in 4 feet? in 5 feet? in 10 feet?

v. How many inches are there in a yard, which is 3 feet? in 2 yards, or 6 feet?

w. How many eggs are there in 7 dozen? in 10 dozen?

123. Examples for the Slate.

	a.	*b.*	*c.*
Multiply	9	9	9
by	10	100	1000
	90	900	9000

Ten nines equal 9 tens (Art. 105), or 90. So 100 nines equal 9 hundreds, or 900; and 1000 nines equal 9 thousands, or 9000. Hence, to multiply by 10, *annex a zero to the multiplicand;* to multiply by 100, *annex two zeros;* to multiply by 1000, *annex three zeros.*

75. Multiply 69 by 10; 69 by 100; 6 by 1000.

76. Multiply 47 by 10; by 100; and add the products.

77. Multiply 8 by 100; by 1000; and add the products.

78. Multiply 9 by 10; by 100; by 1000; and add the products.

79. Multiply 5 by 1000; by 100; by 10; and add the products.

80. Multiply 42 by 10; 268 by 10; 3 by 1000; and add the products.

124. *d.* Multiply 30
by 40
1200

When the multiplicand and multiplier, or either of them, have zeros at the right hand, *disregard the zeros in multiplying, but annex to the product as many zeros as were disregarded.*

	(81.)	(82.)	(83.)	(84.)	(85.)	(86.)
Multiply	40	300	240	710	480	20
by	60	2	20	30	30	400

87. When hay is $ 20 a ton, what must be given for 10 tons? for 100 tons? for 20 tons?

88. There are 20 things in a score. How many things are there in 100 score?

89. What is the cost of 20 rakes at 35 cents each?

90. Mr. Oakes sold 140 bushels of potatoes at 60 cents a bushel. What did he receive for them?

91. How many men working 1 hour would be required to do as much as 80 men working 10 hours?

92. Multiply 23 by 2 and by 30 and add the products.
93. Multiply 42 by 4 and by 50 and add the products.
94. Multiply 56 by 3 and by 20 and add the products.
95. Multiply 61 by 5 and by 50 and add the products.
96. Multiply 72 by 6 and by 50 and add the products.
97. Multiply 37 by 4 and by 70 and add the products.
98. Multiply 36 by 3 and by 40 and add the products.
99. Multiply 95 by 5 and by 80 and add the products.
100. Multiply 48 by 7 and by 60 and add the products.
101. Multiply 29 by 4 and by 30 and add the products.

125. *e.* Multiply 29 by 34.

WRITTEN WORK.

29
34
116 = product by 4.
87 = product by 30.
986 = product by 34.

To multiply by 34, multiply first by 4, then by 30, and add the products.

In multiplying by 30, the product is 870, but it is not necessary to write the zero.

	(102.)	(103.)
Multiply	32	84
by	84	32

If you have made no mistake, the answers to Examples 102 and 103 are equal. Hence, to prove examples in multiplication, *multiply the multiplier by the multiplicand. The two products ought to be equal.*

Multiply and prove the following examples:

(104.)	(105.)	(106.)	(107.)	(108.)	(109.)
34	52	76	48	39	83
× 45	× 37	× 58	× 64	× 25	× 98

110. How many are 67 × 79?
111. How many are 67 × 36?
112. How many are 52 × 45?
113. How many are 68 × 25?
(114.) 94 × 79 = what?
(115.) 83 × 37 = what?
(116.) 79 × 67 = what?
(117.) 94 × 89 = what?

126. *f.* Multiply 145 by 27; 205 by 27.

PROOF.	PROOF.
27	27
145	205
135 = product by 5.	135 = product by 5.
108 = product by 40.	54 = product by 200.
27 = product by 100.	5535 = product by 205.
3915 = product by 145.	

118. Multiply 184 by 47.
119. Multiply 235 by 39.
120. Multiply 605 by 15.
121. Multiply 62 by 134.
122. How many are 583 × 56?
123. How many are 28 × 470?
124. How many are 731 × 13?
125. How many are 33 × 206?

For other examples in Multiplication, see page 71.

127. Applications.

126. At 11 cents a pound, what will a barrel of 269 pounds of sugar cost?

127. If a train of cars runs 453 miles in a day, how many miles will it run in 21 days?

128. If a tailor makes 432 coats in a year, and puts 14 buttons on each coat, how many buttons does he use?

129. Mr. Frost's cow Lily yielded 154 pounds of butter in a year. The butter was sold at 35 cents a pound. How much money did Mr. Frost receive for the butter?

130. A party of 23 persons hired a schooner, paying $3.75 apiece for the use of it. How much did all pay?

131. What must be paid for 5 cheeses weighing 85 pounds each, at 16 cents a pound?

132. Twelve dozen make a gross. Twelve gross make a great gross. How many pens are there in a great gross of pens?

133. A family of 4 boarded in Plymouth for 9 weeks at the rate of $6.50 a week for each person. What was the cost of board for the whole time?

128. Miscellaneous Oral Examples.

Repeat the following tables:

Liquid Measure.	Dry Measure.
4 gills = 1 pint.	2 pints = 1 quart.
2 pints = 1 quart.	8 quarts = 1 peck.
4 quarts = 1 gallon.	4 pecks = 1 bushel.

a. How many quarts are there in one gallon? how many pints? how many gills?

b. What is the cost of a gallon of milk at 6 cents a quart?

c. If a gill of milk is required for a pint of coffee, how many gills are required for a quart? for a gallon?

d. How many quarts are there in 2 gallons and 1 quart?

e. In a bushel there are how many pecks? how many quarts? how many pints?

f. At 10 cents a quart for tomatoes, how many cents will a man receive for 1 bushel and 2 pecks?

g. If I buy 3 bunches of asparagus at 8 cents a bunch, 2 quarts of new potatoes at 10 cents a quart, and some lettuce for 13 cents, how many cents should I pay for them?

h. If what I bought came to 57 cents, and I gave in payment a 1-dollar bill, how much change should I receive in return?

NOTE. To take 57 cents from $1, or 100 cents, take away first 50 cents, and then take 7 cents from the remainder.

i. John was sent to the store with 60 cents to buy 3 pounds of beef at 11 cents a pound, 6 pounds of rhubarb at 2 cents a pound, and 2 bunches of radishes at 5 cents a bunch. How many cents should he bring back?

j. Edith bought 4 yards of cambric at 12 cents a yard and had 9 cents left. How many cents had she at first?

k. 12 inches make a foot and 3 feet make a yard. How many inches make a yard?

l. Measure your height in inches, also measure the distance you can reach with your arms extended, and find the difference.

NOTE. In doing the following examples write the numbers on your slate, if necessary.

m. A traveller bought for his breakfast, 2 sandwiches at 8 cents apiece, a cup of coffee at 10 cents, and 2 pears at 6 cents apiece. What did his breakfast cost him?

n. For his dinner he bought steak for 30 cents, 2 bananas at 8 cents apiece, some pie for 10 cents, and a cup of tea for 5 cents. What did his dinner cost him?

o. How much more did his dinner cost than his breakfast?

p. How much did his breakfast and his dinner both cost?

q. How much change should he receive if he gave in payment for his breakfast and dinner a 1-dollar bill?

r. 3 and 2, multiplied by 2, less 6, multiplied by 3, are how many?

s. 7 times 3, plus 4, plus 5, less 6, less 8, are how many?

t. Take 14, subtract 8, add 3, add 2, multiply by 2.

u. 6, and 4, and 7, and 5, less 12, multiplied by 4 = what?

v. Think of any number less than 8, multiply that number by 5, subtract 3 times the number thought of, add 2, add 4, subtract twice the number thought of. What number have you?

129. Examples for the Slate.

134. In a quire of paper there are 24 sheets. How many sheets are there in 20 quires or a ream?

135. After a collection was taken in church, one box contained $25.34, another $10.17, another $14.21, and another $15.98. How much did all contain?

136. In a basket of eggs I counted 175. 82 of them were turkey's eggs and the rest were hen's eggs. How many hen's eggs were there?

137. Charles started to carry a basket containing 132 eggs, but he fell and broke 17 turkey's eggs and 29 hen's eggs. How many eggs remained whole?

138. Add together all the numbers from 1 to 24.

139. There were landed from a ship 38 boxes of lemons. If each box contained 250 lemons, how many lemons were there in all?

140. In an orchard of 125 trees, the owner set out 37 more, of which 13 died. How many remained alive?

141. If 1 sparrow destroys 235 caterpillars in a day, how many will 6 sparrows destroy in a week?

142. How many panes of glass are there in a house which has 45 windows of 6 panes each, and 4 cellar windows of 3 panes each?

143. A hod-carrier went up and down a ladder 42 times in a day. If the ladder had 30 rounds, how many steps did he take upon it during the day?

144. The children of a school can be arranged in 24 rows, 28 in a row. How many more children would be needed to make 27 rows, 27 in a row?

145. A boy bought a writing-desk for $2.75 and a jack-knife for $1.37. He spoiled his jack-knife in mending his writing-desk and then sold the writing-desk for $3.86. Did he gain or lose by the whole transaction, and how much?

146. A barrel of flour contains 196 pounds. If a pound of flour is put into each loaf of bread, how many loaves may be made from 25 barrels of flour?

147. If 7 persons consume a barrel of flour in 13 weeks, how many persons will consume 20 barrels in 1 week?

148. A man bought 4 bushels of potatoes for $3.50, and sold them at 80 cents a peck. How much did he receive for what he sold? How much did he gain?

149. A grocer paid $5.00 for a box containing 294 oranges; 17 of the oranges were spoilt, and he sold the remainder at 5 cents apiece. Did he gain or lose, and how much?

150. A dealer counted 4 boxes of oranges, and found in the first box 32 dozen and 4; in the second, 35 dozen and 3; in the third, 28 dozen and 11; and in the fourth, 36 dozen and 7. How many oranges were there in all?

SECTION V.

DIVISION.

Division by 2, 3, and 4.

130. How many apples are there on the table?

If you give these 6 apples to some children, giving them 2 apiece, to how many children will you give them?

a. How many 2's, or how many times 2, are there in 6?

131. Finding how many times one number is contained in another is ***dividing.***

132. The number to be divided is the ***dividend.***

133. The number by which we divide is the ***divisor.***

134. The result obtained by dividing is the ***quotient.***

b. Divide 10 by 2, that is, find how many 2's there are in 10.

c. What is the quotient of 18 divided by 2? of 20 divided by 2?

d. If you have 11 apples, to how many children can you give 2 apiece, and how many apples will be left?

135. The part of the dividend left after the equal numbers are taken away is the ***remainder.***

e. How many 2's are there in 13, and what is the remainder?

f. What is the quotient of 28 divided by 4?

136. 28 divided by 4 may be written thus, $28 \div 4$. The sign $\div$ means "divided by."

g. What is the quotient of $36 \div 4$? of $36 \div 3$?

h. What is the quotient of $27 \div 3$? of $22 \div 2$?

i. What is the quotient of $13 \div 3$, and what remains?

137. Drill Exercises.

j. Repeat the multiplication table of 2's.

k. How many 2's in 2? 4? 6? 8? 10? 12? 14? 16? 18? 20?

l. Write the division table in the form given in the margin, from 2 in 2 to 2's in 24.

m. Repeat the division table forward and back.

TABLE.*

2 in 2	1.
2's in 4	2.
2's in 6	3.
Etc.	

Divide by 2 the numbers written below, naming quotients and remainders at sight.

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
n. 2)	2	10	12	4	8	6	16	14	18	24	22	20
o. 2)	3	11	13	5	9	7	17	15	19	25	23	21

138. *p.* Repeat the multiplication table of 3's.

q. Write the division table of 3's from 3 in 3, to 3's in 36.

r. Repeat the table forward and back.

Divide by 3 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
s. 3)	3	15	9	27	6	18	12	24	36	21	33	30
t. 3)	4	17	11	28	8	19	14	25	37	23	34	31

* See Appendix, page 138.

139. *u.* Repeat the multiplication table of 4's.

v. Write the division table of 4's from 4 in 4 to 4's in 48.

w. Repeat the table forward and back.

Divide by 4 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
x.	4) 16	8	4	24	12	28	20	32	40	48	36	44
y.	4) 18	11	7	25	15	30	23	35	43	49	37	45

140. Oral Examples.

a. How many tops at 2 cents each, can be bought for 6 cents? for 10 cents?

b. At 2 cents each, how many tops can be bought for 21 cents, and how many cents will be left?

*Solution.** — As many tops can be bought for 21 cents as there are 2's in 21; which is 10, and 1 remains. *Ans.* 10 tops; 1 cent remains.

c. How many rows of chairs, 2 in a row, can you make with 4 chairs? 12 chairs? 19 chairs?

d. How many quarts are there in 10 pints? in 14 pints? in 24 pints?

e. In one yard there are 3 feet. How many yards are there in 6 feet? in 9 feet? in 10 feet?

f. At 3 cents an hour, how many hours must you work to earn 15 cents? 24 cents? 30 cents?

g. Four quarters make 1 dollar. How many dollars in 8 quarters? in 12 quarters? in 15 quarters?

h. How many gallons are there in 12 quarts? in 14 quarts?

i. How many bushels are there in 4 pecks? in 16 pecks? in 21 pecks?

j. How many pints in 4 gills? in 20 gills? in 30 gills?

k. If a coat can be made from 4 yards of cloth, how many coats can be made from 24 yards? from 29 yards?

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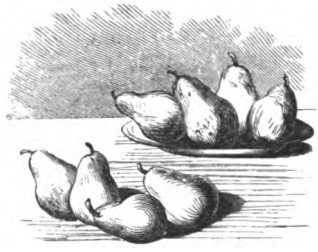
* See Appendix, page 139.

Dividing Numbers into Equal Parts.



141. If 8 pears be divided equally between 2 boys, how many pears will each boy have?

Each boy will have one of the two equal parts into which 8 is divided. We have seen by multiplication that 2 fours are 8, so we know that each boy will have 4 pears.

142. If 1 pear be divided equally between two boys, what will each boy have?

When any number or any thing is divided into two equal parts, the parts are called ***halves.*** When it is divided into three equal parts, the parts are called ***thirds.*** When it is divided into four equal parts, the parts are called ***fourths;*** and so on.

143. Oral Exercises.

a. What is 1 half of 8 apples? of \$ 10? of 14? of 18?

b. What is 1 third of 6 pears? of \$ 9? of 15? of 27?

c. What is 1 fourth of 8 cents? of \$ 16? of 12? of 20?

d. If 12 figs be divided equally between two boys, how many figs will each boy receive?

Solution. — Each boy will receive 1 half of 12 figs, which is 6 figs. *Ans.* 6 figs.

e. How many figs would each boy receive if 12 figs were divided equally among 3 boys? among 4 boys?

f. How many fishes would each person receive if 24 fishes were divided equally among 3 persons? among 4 persons?

g. By what do you divide to find 1 half of a number? 1 third? 1 fourth?

h. How many are $9 \div 3$? 9 tens $\div 3$? 9 hundreds $\div 3$?

i. How many are $8 \div 4$? 8 tens $\div 4$? 8 hundreds $\div 4$?

j. How many are $27 \div 3$? 27 tens $\div 3$? 27 hundreds $\div 3$?

Examples for the Slate.

144. Copy the following examples on your slate. Beginning with the hundreds, *divide the hundreds, the tens, and the units separately,* writing the quotient as it is written in Example **a.**

a.

Divisor 3) 306 *Dividend.*

102 *Quotient.*

In doing Example **a,** say "3's in 3, 1; in 0, none; in 6, 2."

To prove your work, *find the product of the divisor and quotient. This product ought to equal the dividend.*

(1.)	(2.)	(3.)	(4.)	(5.)
2) 842	2) 460	3) 366	3) 906	4) 480

b. **c.**

3) 1230 2) 1418

410 709

In Example **b,** as 3 is larger than 1, begin by dividing 12 hundreds. In Example **c,** begin by dividing 14 hundreds; and when you come to the tens, as 2 is larger than 1, put a zero in the tens' place of the quotient and divide 18 units.

(6.)	(7.)	(8.)	(9.)	(10.)
3) 1569	3) 2118	2) 1612	4) 1620	4) 8360

d.

4) 426—2 *Rem.*

106

In Example **d,** after dividing 26 by 4, there is a remainder of 2, which is written at the right of the dividend.

NOTE. In proving the work, *the remainder must be added to the product of the divisor and quotient.*

(11.)	(12.)	(13.)	(14.)	(15.)
3) 2722	4) 2814	4) 3225	3) 2405	3) 3277

16. If 364 children are marching, 3 in a row, how many rows are there?

17. How many rows would there be, if 364 were marching, 4 in a row?

18. Albert bought 4 canary birds for $8.32. What was the price of each?

19. A father left $8420 to his four children. What was the share of each child?

Division by 5, 6, and 7.

145. Drill Exercises.

a. Repeat the multiplication table of 5's.

b. Write the division table of 5's from 5 in 5 to 5's in 60.

c. Repeat the table forward and back.

Divide by 5 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
d. 5)	10	30	50	25	5	35	55	15	40	60	20	45
e. 5)	13	32	54	26	7	39	57	16	43	62	24	48

146. *f.* Repeat the multiplication table of 6's.

g. Write the division table of 6's from 6 in 6 to 6's in 72.

h. Repeat the table forward and back.

Divide by 6 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12
i. 6)	12	36	60	72	6	42	66	18	48	24	54	30
j. 6)	15	37	64	77	11	44	68	23	49	29	58	34

147. *k.* Repeat the multiplication table of 7's.

l. Write the division table of 7's from 7 in 7 to 7's in 84.

m. Repeat the table forward and back.

Divide by 7 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
n. 7)	28	49	7	63	77	14	56	70	21	84	35	42
o. 7)	31	54	11	69	82	17	62	72	23	88	36	46

148. Oral Examples.

p. If a sack can be made out of 5 yards of cloth, how many sacks can be made out of 25 yards? 30 yards? 34 yards?

q. 6 feet make a fathom. How many fathoms deep is it to a rock that is under water 24 feet? 36 feet? 48 feet?

r. If an anchor sinks 59 feet, how many fathoms does it sink and how many feet besides?

s. How many weeks and how many days over are there in 25 days? in 37 days? in 60 days? in 72 days?

t. What is 1 fifth of 25 days? 1 sixth of $36? 1 seventh of 84 cents?

u. Charles bought a velocipede for $15, and paid 1 fifth of the price every month. How many dollars did he pay a month? How many months did it take him to pay the whole?

149. Examples for the Slate.

(20.)	(21.)	(22.)	(23.)
5) 1520	5) 3541	6) 1809	7) 6329

a.

5) 173 – 3

34

In dividing 17 (tens) by 5, we have 3 (tens) for the quotient, and 2 tens remain. We write 3 under the line in the tens' place, and unite the 2 tens with the 3 units of the dividend, making 23 units.

5's in 23, 4 and 3 remain, which we express as in the written work above. *Ans.* 34 and 3 remain.

(24.)	(25.)	(26.)	(27.)	(28.)	(29.)
5) 184	5) 283	6) 728	7) 1230	6) 326	7) 8142

30. How many sheep at $6 apiece can be bought for $159, and what will remain?

31. In 365 days how many weeks, and what remains?

32. A milkman received $86.45 for milk which he sold at 7 cents a quart. How many quarts did he sell?

33. How many settees will be required to accommodate 1000 persons if 5 are seated on each settee?

34. Five men hired a boat for $12.75, agreeing to divide the cost equally. What was the share of each?

35. Six men shared equally a profit of $7464. What did each receive?

36. What is 1 fifth of 1375 oxen? 1 seventh of $4529?

Division by 8, 9, and 10.

Drill Exercises.

150. *a.* Repeat the multiplication table of 8's.

b. Write the division table of 8's from 8 in 8 to 8's in 96.

c. Repeat the table forward and back.

Divide by 8 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)
d. 8)	16	40	64	80	8	48	88	24	56	96	32	72
e. 8)	22	45	66	87	13	54	89	31	58	99	36	75

151. *f.* Repeat the multiplication table of 9's.

g. Write the division table of 9's from 9 in 9 to 9's in 108.

h. Repeat the table forward and back.

Divide by 9 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
i. 9)	45	90	36	72	54	99	18	63	81	27	108
j. 9)	51	92	42	76	61	102	21	65	88	32	112

152. *k.* Repeat the multiplication table of 10's.

l. Write the division table of 10's from 10 in 10 to 10's in 120.

m. Repeat the table forward and back.

Divide by 10 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
n. 10)	20	31	45	60	57	73	88	90	108	123	110

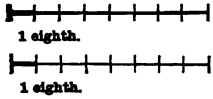
Oral Examples.

153. *o.* Divide 9 yards of ribbon equally among 8 girls. Divide 10 yards.

If 9 yards are divided among 8 girls, each girl will receive 1 yard, and there will still be 1 yard to be divided.

If 1 yard is divided among 8 girls, each girl will receive 1 eighth of a yard. So each girl will have 1 and 1 eighth yards.

If 10 yards are divided among 8 girls, after giving 1 yard to each girl, there will be 2 yards to be divided, which will give 2 eighths of a yard apiece. So each girl will have 1 and 2 eighths yards.



p. 8 men bought a fishing-net for $21. What part of the price should each pay? How many dollars?

q. What is 1 eighth of 20? of 23? of 34?

r. Divide 41 rods of land into 8 equal parts; divide 52 rods; divide 78 rods.

s. If a party of 10 men catch 56 pounds of fish, what is the share of each?

154. Examples for the Slate.

a. Divide $6530 by 8.

WRITTEN WORK.

8) $6530 − $2.

$816

8) $6530

$816⅜

In the second form of the written work of this example, the last remainder, $2, is divided, giving a quotient of 2 eighths, which is written thus, ⅜. *Ans.* $816⅜.

Divide in the same way the following examples:

(37.)	(38.)	(39.)	(40.)
8) 3297	8) 4542	8) 4125	8) 6321
(41.)	(42.)	(43.)	(44.)
9) 7234	9) 1928	9) 1234	9) 7361
(45.)	(46.)	(47.)	(48.)
8) 5187	9) 1904	10) 8761	10) 3205

49. Mr. Eustis bought a house for $3824 and paid 1 eighth of it every year till it was paid for. How much did he pay a year?

50. Mr. Clark teaches 9 months in the year at a salary of $1900. What is that a month?

51. Miss Alvord teaches 10 months in the year at a salary of $1275. What is that a month?

52. If it costs a family of 8 persons $1535 a year to live, what is the average cost for 1 person?

Division by 11 and 12.

Drill Exercises.

155. ***a.*** Repeat the multiplication table of 11's.

b. Write the division table of 11's from 11 in 11 to 11's in 132.

c. Repeat the table forward and back.

Divide by 11 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
d. 11)	22	33	66	88	110	44	77	121	55	99	132
e. 11)	25	37	71	95	118	53	81	122	65	101	141

156. ***f.*** Repeat the multiplication table of 12's.

g. Write the division table of 12's from 12 in 12 to 12's in 144.

h. Repeat the table forward and back.

Divide by 12 the numbers written below:

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
i. 12)	36	24	96	144	132	48	120	60	108	84	72
j. 12)	42	31	101	152	141	52	122	71	113	93	73

157. Oral Examples.

k. At 12 cents apiece, how many melons can be bought for 24 cents? for 40 cents? for 50 cents?

l. It takes Mary 60 hours to crochet a jacket. How many days will it take her if she works 12 hours a day?

m. How many days will it take Mary to crochet the jacket if she works 11 hours a day?

n. How many feet are there in 12 inches? in 24 inches? in 18 inches? in 96 inches? (See Example *u*, page 44.)

o. When 10 yards of silk can be bought for $15, what is the price of 1 yard?

p. There are 12 months in a year. In 36 months how many years? in 30 months? in 42 months?

q. What is 1 eleventh of \$22? of \$44? of \$66?
r. What is 1 eleventh of \$27? of \$51? of \$72?
s. What is 1 twelfth of 36¢? of 60¢? of 75¢?
t. What is 1 twelfth of 35 days? of 100 days?

158. Examples for the Slate.

Find how many times \$11 is contained in the sums of money written below:

(53.)	(54.)	(55.)	(56.)
\$11) \$5602	\$11) \$6281	\$11) \$3406	\$11) \$4987

Find 1 twelfth of the sums of money written below:

(57.)	(58.)	(59.)	(60.)
12) \$7681	12) \$3908	12) \$85.29	12) \$50.03

For other examples in Division by numbers no greater than 12, see page 71.

159. *a.* Divide 1698 by 32.

WRITTEN WORK.

32) 1698 ($53\frac{2}{32}$
160
98
96
2

In this example, since 32 is a larger number than 16, we first divide 169 (tens) by 32.

There are about as many 32's in 169 as there are 3's in 16, which is 5. We write 5 for the tens of the quotient, at the right of the curved line. Multiplying 32 by 5 (tens), we have 160 (tens). Subtracting 160 from 169, we find 9 tens remain. Uniting with these 9 tens the 8 units of the dividend, we have 98 units.

32's in 98, 3 and 2 units remain. We write 3 for the units of the quotient. Dividing the remaining 2 by 32, we have 2 thirty-seconds, and the entire quotient is $53\frac{2}{32}$.

Divide in the same way the following examples:

(61.)	(62.)	(63.)	(64.)
21) 1164 (	42) 1984 (	44) 3568 (	51) 4897 (
(65.)	(66.)	(67.)	(68.)
33) 2998 (	62) 4528 (	67) 2363 (	41) 3784 (

69. How many are $394 \div 34$?
70. How many are $538 \div 47$?
71. How many are $784 \div 26$?
72. How many are $892 \div 75$?
73. Divide 3949 by 84.
74. Divide 7009 by 51.
75. Divide 7958 by 68.
76. Divide 9876 by 92.

160. *b.* Divide 1792 by 24.

WRITTEN WORK.

24) 1792 (74$\frac{16}{24}$
168
112
96
16

In Example *b*, when we say that there are about as many 24's in 179 as 2's in 17, we find 8 for the first term of the quotient; but on multiplying we get 192, a number too large to subtract from 179. Therefore 8 is too large for the first term of the quotient, and we try 7, as shown in the work above.

77. How many are $1443 \div 74$?
78. How many are $2938 \div 77$?
79. How many are $8642 \div 46$?
80. How many are $4672 \div 58$?
81. Divide 9278 by 87.
82. Divide 2876 by 43.
83. Divide 6444 by 96.
84. Divide 5105 by 64.

To divide by 10, 100, or 1000.

161. *c.* In 1456, how many tens and what remains?

Ans. 145 tens and 6 remain.

d. In 1456, how many hundreds and what remains? how many thousands and what remains?

e. Divide 1456 by 10; by 100; by 1000.

$1456 \div 10 = 145.6$
$1456 \div 100 = 14.56$
$1456 \div 1000 = 1.456$

To divide by 10, *put a point in the dividend one place to the left;* to divide by 100, *put a point two places to the left;* to divide by 1000, *put a point three places to the left.*

In Example *e*, the first answer is 145 and 6 tenths; the second is 14 and 56 hundredths; and the third is 1 and 456 thousandths.

85. Divide 3456 by 100.
86. Divide 252 by 100.
87. Divide \$3000 by 10.
88. Divide \$8100 by 10; by 100.
89. Divide \$1649 by 100; by 10.
90. Divide \$4930 by 1000.

To divide United States Money, carrying the Division to Cents.

162. *f.* Divide \$34.87 by 32. *g.* Divide \$45 by 32.

WRITTEN WORK.

f.

32) \$34.87 (\$1.08$\frac{31}{32}$
32
287
256
31

g.

32) \$45 (\$1.40$\frac{20}{32}$
32
1300
128
20

In Example *f*, the dividend has 2 places for cents, so in the quotient we mark off 2 places for cents.

In Example *g*, after the dollars are divided there is a remainder of \$13, which equals 1300 cents. 1300 cents divided by 32 gives $40\frac{20}{32}$ cents, and the whole quotient is \$1.40$\frac{20}{32}$.

In doing the following examples, *whenever a remainder is dollars, annex two zeros and continue the division to cents.*

91. Divide \$48.40 by 15.
92. Divide \$82.36 by 32.
93. Divide \$97.48 by 63.
94. Divide \$99.19 by 42.
95. Divide \$49.32 by 54.
96. Divide \$85.12 by 81.
97. Divide \$48 by 14.
98. Divide \$28 by 32.
99. Divide \$42 by 34.
100. Divide \$84 by 41.
101. Divide \$4 by 25.
102. Divide \$8 by 27.

163. Applications.

103. How many piles of 20 half-dollars each, can you make with 348 half-dollars?

104. How many piles of 40 quarter-dollars each, can you make with 583 quarters?

105. Mr. Goss planted 1296 hills of corn in rows of 27 hills each. How many rows did he plant?

106. If Mr. Goss had planted the same number of hills in 16 rows, how many hills would he have had in a row?

107. Mr. Green earned \$28.68 in a month and spent 1 twelfth of what he earned for books. How much did he spend for books?

108. A man bought a house for $2175, agreeing to pay 1 tenth of the price every year till it was paid for. How much did he agree to pay each year. How many years would it take him to pay the debt?

109. How many barrels can be filled with 2000 pounds of flour, each barrel containing 196 pounds?

110. A load of potatoes weighing 742 pounds was put into bags of 2 bushels each. If each bushel weighed 60 pounds, how many full bags were there, and how many pounds remained?

111. How many dollars are there in 2762 cents? in 5648 cents?

112. If a man walks 25 miles a day, how many days will it take him to walk 1000 miles?

113. There are 5280 feet in a mile. How many times must a wagon-wheel turn in going a mile if it is 14 feet round the rim?

For other examples in Division, see page 71.

Miscellaneous Oral Examples.

164. *a.* Ira sells cherries at 10 cents a quart. How many quarts must he sell to receive 50 cents? 90 cents? $1?

b. Carl sells berries at 9 cents a quart. How many quarts must he sell to get enough money to buy a hat worth 70 cents and a slate worth 11 cents?

c. Jane makes paper-bags at 8 cents an hour. How much will she earn in half an hour? in 4 and 1 half hours?

d. How many peck baskets will it take to hold 8 quarts of plums? 32 quarts? 28 quarts?

e. How many times must a quart measure be filled to measure out a bushel and a half of cranberries?

f. How many quart cans will it take to hold 8 and 1 half gallons of milk?

g. How many times must you pick a pint dish full of berries to fill a basket that holds a peck? that holds a peck and 2 quarts?

h. How many pounds of beef at 12 cents a pound will be required to pay for 9 quarts of cherries at 8 cents a quart?

i. If $81 is paid for 9 weeks' labor, what is paid for 4 weeks' labor? for 7 weeks'? for 12 weeks'?

j. If 6 yards of muslin cost 60¢, what will 7 yards cost? 10 yards? 12 yards?

k. If 7 persons consume a barrel of sugar in 9 months, how long will it last 1 person? How long will it last 8 persons?

l. If a family of 3 persons consume a barrel of flour in 6 months, how long will it last if 2 persons are added to the family?

165. Repeat the following table of

Time.

60 seconds	= 1	minute.
60 minutes	= 1	hour.
24 hours	= 1	day.
7 days	= 1	week.
52 weeks and 1 day or 365 days	= 1	year.
100 years	= 1	century.

a. How many seconds are there in half a minute?

b. How many minutes are there in half an hour? in a quarter of an hour? in an hour and a quarter? in an hour and a half?

The word "day" as used in the table denotes the interval of time between one midnight and the next. As used in the following example, it denotes the interval of time between sunrise and sunset.

c. When the sun rises at six o'clock and sets at six o'clock, how long is the day? how long is the night?

d. When the days are 11 hours long, how long are the nights?

e. If a person is 84 years old, how many years longer must he live to be a century old?

There are 12 months in the year, with days as follows:

"Thirty days hath September,
April, June, and November;
All the rest have thirty-one,
Except the second month alone,
To which we twenty-eight assign,
Till leap-year gives it twenty-nine."

f. Name the months that have 30 days.

g. What month has 28 days? How many days has it in leap-year?

h. How many days has each of the other months?

i. The winter months are December, January, and February. How many days are there in winter?

j. The spring months are March, April, and May. How many days are there in spring?

k. The summer months are June, July, and August. How many days are there in summer?

l. The autumn months are September, October, and November. How many days are there in autumn?

166. Repeat the following tables:

Weight.
16 ounces = 1 pound.
2000 pounds = 1 ton.

Numbers.
12 units = 1 dozen.
12 dozen = 1 gross.
20 units = 1 score.
12 gross = 1 great gross.

m. If an ounce of chocolate is used at each meal, for how many meals will 2 pounds of chocolate serve?

n. If 200 pounds of beef are put into a barrel, how many barrels will it take to hold a ton?

o. How many dozen buttons are there on a card that contains half a gross? How many buttons on the card?

p. The allotted age of man being "threescore years and ten," how old is a man who has lived 17 years beyond the allotted age?

167. Miscellaneous Examples for the Slate.

114. A farmer has 45 sheep and 18 lambs. How many more sheep has he than lambs? How many of both has he?

115. A flour-dealer sold 28 barrels of flour to one man, 33 to another, 17 to a third, and then had 115 barrels left. How many barrels had he at first?

116. If 6 sheets of paper are required to make a book of 192 pages, how many pages are printed on one sheet?

117. At 4¢ a dozen for hooks, what will it cost for hooks for 4 closets having 18 hooks in each, and 2 closets having 30 hooks in each.

118. Belle gave a half-dollar, a dime, and 3 cents for a fan, and had $2.58 left. How much had she at first? How many cents did the fan cost?

119. A newsboy sold 16 Journals at 3¢ apiece, 23 Heralds at 2¢ apiece, 13 Advertisers at 4¢ apiece, and 18 Transcripts at 4¢ apiece. What did he receive for all?

120. At $2 a pound for worsted, what will 1 ounce cost?

121. Kate bought a pound and a quarter of worsted at $2 a pound, a thimble for 85 cents, and a crochet-needle for 7 cents. How much did she pay for all?

122. Mr. Hubbard bought 6 gross of lead-pencils at $1.80 a gross. What did he pay for the lot?

123. If 1 gross of pencils can be bought for $1.80, what is that per dozen?

124. Here are the charges for Olive's bonnet. What did it cost?

125. Olive's bonnet cost $1.89 more than Mary's. What did Mary's cost?

126. What was the cost of both bonnets, and 2 scarfs at $1.62 apiece?

CHARGES.

Silk	$1.87
Velvet	1.12
Ribbon	.63
Lace	.38
Frame and making	1.60
Amount	$

127. If 196 pounds of flour cost $9.80, what cost 50 pounds?

128. At $3.20 a bushel for cranberries, what cost 9 quarts?

129. I sent a message of 34 words by telegraph, paying 25 cents for the first 10 words, and for each remaining word 2 cents. What did the message cost?

130. If a horse-car makes 12 trips a day, and takes on the average 24 passengers at 5 cents apiece, how much money does the conductor take in a day?

131. Dr. Lamb caught 13 and a half pounds of trout, worth 36 cents a pound. What was the lot worth?

132. When oysters are 38 cents a quart and shad 12 cents a pound, what shall I pay for a pint of oysters and 3 and a half pounds of shad?

133. If a horse-shoer puts 8 nails in each shoe and has used up 1376 nails, how many shoes has he set? How many horses has he shod, each horse being shod all round?

134. If a watch loses 7 seconds a day, how many seconds will it lose in a year? How many minutes?

135. At 4 cents a dozen for clothes-pins, what will 300 clothes-pins cost?

136. If you buy 6 papers containing 25 needles each, for 25 cents, how many needles do you get for 1 cent?

137. At 62 cents a bushel, what will 5 and a half bushels of potatoes cost?

138. A clothier bought 12 coats for $13.92, and sold them for $1.50 apiece. What did he gain on the lot?

139. If two gallons of lemonade are made with 14 lemons costing 36¢ a dozen, and 2 pounds of sugar costing 11¢ a pound, what is the cost of a gallon of lemonade? of a quart? of one glass holding half a pint?

140. If a family consume 2 quarts of milk a day at 7 cents a quart from October 1 to April 1, and 3 quarts a day at 6 cents a quart from April 1 to October 1, what is the cost of milk for the year?

141. A man bought 350 cords of wood for $962.50. If he sold 100 cords at $2.65 a cord, 175 cords at $3.20 a cord, and the rest at $3.33 a cord, did he gain or lose, and how much?

142. How many pins are there in a paper having 12 rows, and 30 pins in a row? If you can buy the pins for 18 cents, how many pins do you get for 1 cent?

143. Which is cheaper, to buy 360 pins for 18 cents, or a paper containing 12 rows of 20 pins each, for 10. cents?

144. As Mark rode to his uncle's, he watched one of the wagon-wheels and found it turned 550 times. The wheel measured 13 feet round. How many feet did he ride? How many miles?

145. Alfred can walk around a certain pond in 9 minutes, and Bertram can walk around in 8 minutes. How many times can Alfred walk around while Bertram walks around 36 times?

146. How many feet is it from the first floor of a house to the attic, which is reached by 2 flights of stairs, the first having 16 steps, and the second 13 steps, each step being 8 inches high, and 12 inches making 1 foot?

147. Here is a bill of goods bought by Mr. James Springer of R. H. White & Co. Find what all the goods cost, and make a copy of the bill upon paper.

Boston, Oct. 7, 1878.

Mr. James Springer **Bought of R. H. WHITE & CO.**

3 yards velvet	*@*	*$3.50*	$	
4 " cashmere	*"*	*.90*		
2 " silk	*"*	*3.00*		
8 towels	*"*	*.38*		
1 hat, $4.25; 1 cane, $1.25				
1 valise, $2.25; 1 purse, 62¢				
1 dozen pairs socks			*2*	*75*
			$	

Received payment,

R. H. White & Co.

168. DRILL TABLE No. 2.

Simple Numbers.

Examples.	A	B	C	D	E	F	M *q r*	N *s t u*	O *v w x y*	P
1.	4	16	25	39	57	65	28	987	6909	7
2.	7	19	32	43	51	67	49	995	2985	3
3.	2	22	29	38	55	61	24	872	4360	5
4.	6	14	35	46	49	66	54	327	2943	9
5.	3	20	26	37	54	68	36	436	2616	6
6.	8	13	34	44	56	62	72	216	1728	8
7.	5	17	30	40	50	70	45	543	1086	2
8.	7	23	27	45	58	63	56	842	5894	7
9.	11	15	33	41	52	69	88	471	1884	4
10.	9	21	31	47	59	64	27	174	1044	6
11.	10	18	28	42	53	71	66	862	3448	4
12.	12	24	36	48	60	72	96	854	1708	2
13.	G	H	I	J	K	L	81	534	3738	7
14.	74	90	99	115	124	135	16	291	2619	9
15.	80	93	97	110	121	139	39	487	3896	8
16.	73	86	98	114	126	134	58	390	1950	5
17.	82	95	103	118	130	143	94	876	2628	3
18.	75	85	101	109	128	138	63	912	5472	6
19.	79	91	108	116	122	133	42	396	2772	7
20.	81	87	107	113	131	142	33	483	1932	4
21.	76	89	100	111	125	136	95	235	1880	8
22.	78	94	106	117	123	141	57	387	1161	3
23.	83	88	104	112	127	137	18	897	4485	5
24.	77	92	102	119	129	140	48	624	5616	9
25.	84	96	108	120	132	144	57	246	1476	6

169. Exercises upon the Table.

Examples for the Slate.

Multiply N

21. By 2.
22. By 3.
23. By 4.
24. By 5.
25. By 6.
26. By 7.
27. By 8.
28. By 9.
29. By 11.
30. By 12.

Multiply M

31. By 23.
32. By 54.
33. By 76.
34. By 98.

Divide N

35. By 2.
36. By 3.
37. By 4.
38. By 5.
39. By 6.
40. By 7.
41. By 8.
42. By 9.
43. By 11.
44. By 12.

Divide O

45. By 2.
46. By 3.
47. By 4.
48. By 5.
49. By 6.
50. By 7.
51. By 8.
52. By 9.
53. By 11.
54. By 12.

55. Divide O by P.
56. Divide O by M.
57. Divide O by N.

Miscellaneous.

58. What will P yards of cloth cost at M cents a yard?

59. If a person buys goods for N cents, and gives in payment a 10-dollar bill, what should he receive in return?

60. At P cents each, how many pears can you buy for N cents?

61. Divide O by P and add N to the quotient.

62. If 7 men can do a piece of work in M days, in how many days can 8 men do it?

Oral Practice.

63. Multiply q by r and add s to the product.

64. Multiply r by s and add t to the product.

65. Multiply s by t and add u to the product.

66. Divide A and B each by 2.

67. Divide A, B, and C each by 3.

68. Divide B to D each by 4.

69. Divide B to E each by 5.

70. Divide B to F each by 6.

71. Divide B to F each by 7; by 8; by 9; by 10; by 11; by 12.

72. Divide G by 7.

73. Divide G and H each by 8.

74. Divide G, H, and I each by 9.

75. Divide G to J each by 10.

76. Divide G to K each by 11.

77. Divide G to L each by 12.

78. Multiply q by r and divide the quotient by s.

SECTION VI.

Numbers from Ten Thousand to Millions.

170. Count by tens from ten to a hundred.

Count by ten-thousands from ten thousand to a hundred thousand.

Ten thousand is written 10000 or 10,000.
Twenty thousand is written 20000 or 20,000.
And so on.

How many figures are needed to write ten-thousands? Which figure shows how many ten-thousands there are?

171. Read the following:

a. 10,000.	*c.* 75,000.	*e.* 98,000.	*g.* 48,007.
b. 40,000.	*d.* 17,000.	*f.* 36,549.	*h.* 93,080.

Turn to page 77, and read the numbers written in column *C*.

172. Write in figures the numbers made up of ten-thousands from ten thousand to ninety thousand.

Write in figures the following:

1. Thirty-four thousand, two hundred seventy-five.
2. Eighty-nine thousand, thirty-six.
3. Fifty-four thousand, five hundred two.
4. Fifteen thousand, twelve.

Let the teacher dictate other numbers for the pupil to write.

173. Count by hundred-thousands from one hundred thousand to ten hundred thousand.

One hundred thousand is written 100000 or 100,000.
Two hundred thousand is written 200000 or 200,000.
And so on.

How many figures are needed to write hundred-thousands? Which figure shows how many hundred-thousands there are?

174. Read the following:

i. 400,000.	*k.* 527,810.	*m.* 324,517.	*o.* 407,086.
j. 438,261.	*l.* 906,224.	*n.* 872,239.	*p.* 370,805.

Turn to page 77, and read the numbers written in column *B*.

175. Write in figures:

5. Five hundred sixty-eight thousand, four hundred.
6. Four hundred twenty thousand, twenty-six.
7. Nine hundred nine thousand, nine.
8. Eight hundred forty-one thousand, five hundred three.

Let the teacher dictate other numbers for the pupil to write.

176. The figures in the fourth, fifth, and sixth places taken together form a group called the ***thousands' group,*** while the figures in the first, second, and third places form a group called the ***units' group.*** The figures in the units' group express units, tens, and hundreds of units, while the figures in the thousands' group express units, tens, and hundreds of thousands. In writing numbers, *separate the groups of figures by a comma,* as shown in Articles **173** and **174.**

177. Beyond the second group are higher groups. A third group of figures, expressing units, tens, and hundreds of ***millions,*** is shown in the following

TABLE.

9th. Hundred-millions.	8th. Ten-millions.	7th. Millions.		6th. Hundred-thousands.	5th. Ten-thousands.	4th. Thousands.		3d. Hundreds	2d. Tens.	1st. Units.	
7	2	5	,	3	2	8	,	7	8	3	*Figures.*
3d group.				*2d group.*				*1st group.*			
Millions.				**Thousands.**				**Units.**			

How many figures are needed to write millions? ten-millions? hundred-millions?

Which figure shows how many millions there are? which shows how many ten-millions there are? which shows how many hundred-millions there are?

The number written in the table is read

"Seven hundred twenty-five million, three hundred twenty-eight thousand, seven hundred eighty-three."

178. Repeat the following table:

Ten ones (or units)	**make a ten.**
Ten tens	**make a hundred.**
Ten hundreds	**make a thousand.**
Ten thousands	**make a ten-thousand.**
Ten ten-thousands	**make a hundred-thousand.**
Ten hundred-thousands	**make a million.**

179. Read the following:

q. 2,684,500.	*t.* 8,109,019.	*w.* 1,326,709.
r. 9,275,405.	*u.* 4,414,893.	*x.* 5,070,890.
s. 3,118,184.	*v.* 6,005,928.	*y.* 7,654,321.

180. Turn to page 76, and write in figures the numbers given in column *A*.

Let the teacher dictate other numbers to millions for the pupil to write.

181. Miscellaneous Examples for the Slate.

9. How much must be paid for 625 acres of land, at $175 an acre?

10. It is about 25,000 miles round the earth. How many days would it take a person to travel this distance if he travelled 40 miles a day?

11. Mr. Gaines owned real estate valued at $56,000. Of this he sold land worth $5,000 and a house worth $6,750. What was the value of what he had left?

12. At 7 cents a pound, how many cents will a dealer receive

for 50 boxes of soap each containing 72 pounds? How many dollars will he receive?

13. How many square miles are there in the United States, the Northern Lake region containing 484,339 square miles; the Atlantic Slope, 304,530 square miles; the Gulf region, 1,683,303 square miles; and the Pacific Slope, 854,314 square miles?

14. In 1865 Massachusetts had 1,267,031 inhabitants, and in 1875 she had 1,651,912 inhabitants. What was the increase in ten years?

15. How many seconds are there in the month of January?

16. When Charles is 11 years old, how many hours has he lived, if in that time there have been 3 leap-years?

17. The great bell at Moscow weighs 448,000 pounds. How many tons does it weigh?

18. In one part of her orbit the moon is 224,000 miles from us. If a cannon-ball moves at the rate of 16 miles a minute, in how many minutes will it move through this distance?

19. In 2,468 cocks of hay, averaging 78 pounds each, how many pounds? How many tons?

20. A man bought 42 bushels of potatoes for $7.50, and sold them for 80 cents a peck. How much did he receive for them? How much did he gain?

21. At 4 cents apiece, what will it cost for slats to fence a lot of land having 4 sides each 250 feet long, 5 slats being required for each foot of length?

22. What will be the cost of ties at 35 cents each, to build a mile of railroad, there being 6 ties to a rod and 320 rods in a mile?

23. A farmer bought 5,960 feet of boards at $0.03 a foot, and gave in payment cheese at $0.12 a pound. How many pounds did it take?

24. How many are 3,687,543 + 245,871 + 3,684 + 932,185?

25. From 4,357,859 + 248,946 take 2,783,947.

26. Multiply 7,285 by 394 and by 207.

27. Divide 4,893,683 by 14 and by 25.

182. DRILL TABLE No. 3.

Simple Numbers.

Ex.	A
1.	Five million, eighty-one thousand, two hundred six.
2.	One million, five hundred seven thousand, forty-one.
3.	Eight million, three hundred twenty-one thousand, forty.
4.	Five million, twenty-nine thousand, three hundred.
5.	Four million, seven hundred thousand, four hundred four.
6.	Three million, forty thousand, two hundred thirty-four.
7.	Eight million, six hundred ten thousand, ninety-one.
8.	Nine million, five hundred thirty thousand, seven.
9.	Six million, one hundred sixteen thousand, six hundred.
10.	Two million, seventy-nine thousand, one hundred eighty.
11.	Five million, eighty-four thousand, four hundred nine.
12.	Three million, nine hundred thousand, one hundred two.
13.	Eight million, seven hundred six thousand, fifty-nine.
14.	One million, forty-eight thousand, two hundred eight.
15.	Nine million, eight thousand, three hundred twenty.
16.	Four million, twenty-three thousand, five.
17.	Five million, seven hundred sixty-eight.
18.	Eight million, seventy-two thousand, eighty-nine.
19.	Three million, five hundred seven thousand.
20.	Two million, three hundred forty-eight thousand, eight.
21.	Nine million, twelve thousand, six hundred ninety.
22.	Seven million, seven thousand, seven hundred.
23.	Six million, fifty thousand, two hundred ninety-one.
24.	Four million, five hundred eighty-eight.
25.	Seven million, seven hundred thousand, four hundred six.

DRILL TABLE No. 3

(*continued*).

Examples.	B	C D
1.	987 449	2 5 317
2.	905 788	4 5 995
3.	679 435	3 7 872
4.	369 663	8 8 324
5.	153 967	7 5 436
6.	888 455	5 7 216
7.	806 399	9 4 503
8.	917 517	6 6 842
9.	927 931	1 9 471
10.	953 205	6 7 174
11.	667 871	2 6 862
12.	728 431	9 6 354
13.	845 219	2 9 534
14.	144 868	1 6 111
15.	225 189	5 4 197
16.	199 598	4 5 490
17.	941 529	8 0 876
18.	795 721	3 4 902
19.	734 256	7 6 396
20.	323 583	5 6 484
21.	824 174	4 9 235
22.	769 416	7 4 387
23.	108 824	6 2 897
24.	872 892	3 8 624
25.	444 764	8 7 476

183. Exercises upon the Table.

Addition.

79. Write A in figures.
80. Add A, B, and C.
81. Add in A from 1 to 5; 2 to 6, etc.
82. Add in B from 1 to 8; 2 to 9, etc.
83. Add in C from 1 to 11; 2 to 12, etc.

Subtraction.

84. From A take B.
85. From B take C.
86. From A take C.
87. From 1,001,001 take B.
88. From 9,900,600 take A.

Multiplication.

Multiply B

89. By 2. *92.* By 5. *95.* By 8.
90. By 3. *93.* By 6. *96.* By 9.
91. By 4. *94.* By 7. *97.* By 10.

Multiply D

98. By 12. *101.* By 67. *104.* By 168.
99. By 34. *102.* By 78. *105.* By 312.
100. By 57. *103.* By 87. *106.* By 970.

Division.

Divide A

107. By 2. *110.* By 5. *113.* By 8.
108. By 3. *111.* By 6. *114.* By 9.
109. By 4. *112.* By 7. *115.* By 11

Divide B

116. By 17. *119.* By 904.
117. By 45. *120.* By 586.
118. By 198. *121.* By C.
122. Divide A by B.
123. Divide A by C.

SECTION VII.

FACTORS.

184. What numbers multiplied together will make 6?

Ans. 2 and 3, also 1 and 6; thus, $2 \times 3 = 6$, and $1 \times 6 = 6$.

A number that may be used as multiplicand or multiplier to make another number is a ***factor*** of that number. Thus, 2 and 3 are factors of 6; so also are 1 and 6.

a. Name two factors of 4; of 9; of 12; of 15.

185. Name some factors of 8 besides 8 and 1.

A number that has other factors besides itself and 1 is a ***composite*** number. Thus, 8 is a composite number.

186. Name the factors of 5.

A number that has no other factors besides itself and 1 is a ***prime*** number. Thus, 5 is a prime number.

187. Exercises.

b. Which of the following numbers are prime and which are composite: 6? 7? 8? 10? 11? 12? 13? 14?

c. Write the composite numbers from 1 to 30; from 30 to 50.

d. Write the prime numbers from 1 to 30; from 30 to 50.

188. ***e.*** Name any factors of 12 that are prime numbers. Name any that are not prime.

A factor that is a prime number is a ***prime factor.*** Thus, 2 and 3 are prime factors of 12.

189. *A composite number equals the product of all its prime factors.* Thus, $12 = 2 \times 2 \times 3$.

190. Exercises.

Separate the following numbers into their prime factors and write the results, thus: $6 = 2 \times 3$; $8 = 2 \times 2 \times 2$, and so on.

f. 6.	*h.* 10.	*j.* 18.	*l.* 28.	*n.* 40.
g. 8.	*i.* 16.	*k.* 20.	*m.* 30.	*o.* 45.

191. *p.* Write all the prime factors of 18 and of 24, and name the numbers that are factors of both.

Ans. { The prime factors of 18 are **2**, 3, and **3**.
The prime factors of 24 are **2**, 2, 2, and **3**.
The factors of both are **2**, **3**, and their product **6**.

192. A number that is a factor of two or more numbers is a ***common factor*** of those numbers. Thus, 2, 3, and 6 are common factors of 18 and 24.

q. Which of the numbers 2, 3, and 6 is the greatest common factor of 18 and 24?

193. Exercises.

r. Name a common factor of 8 and 12; of 10 and 15.

s. Name all the common factors of 18 and 12.

t. Name the ***greatest common factor*** of 18 and 12.

What is the greatest common factor

u. Of 9 and 12?	*w.* Of 8 and 12?	*y.* Of 6 and 18?
v. Of 12 and 15?	*x.* Of 18 and 20?	*z.* Of 24 and 36?

Multiples.

194. *a.* Name some numbers which are made by using 2 as a factor. *Ans.* 2, 4, 6, 8, etc.

A number made by using another number as a factor is a ***multiple*** of the number so used. Thus, the numbers 2, 4, 6, and 8 are multiples of 2.

195. *b.* Write the multiples of 3 and of 4 to 24.

Ans. { Multiples of 3 are 3, 6, 9, **12**, 15, 18, 21, **24.**
{ Multiples of 4 are 4, 8, **12**, 16, 20, **24.**

c. Which of these numbers are multiples of both 3 and 4?

A number that is a multiple of two or more numbers is a ***common multiple*** of those numbers. Thus, 12 is a common multiple of 3 and 4; so also is 24.

d. Which of these numbers, 12 and 24, is the least common multiple of 3 and 4?

196. Exercises.

e. Write all the multiples of 2 and of 3 to 30.

f. Which of the numbers you have written are common multiples of 2 and 3?

g. Which is the least common multiple of 2 and 3?

h. Write all the multiples of 4 and of 5 to 40.

i. Which of the numbers you have written are common multiples of 4 and 5?

j. Name the least common multiple of 4 and 5.

Name any common multiple

k. Of 3 and 4; of 3 and 9; of 2, 3, and 4.

l. Of 2 and 5; of 4 and 9; of 5, 10, and 15.

m. Of 5 and 7; of 8 and 12; of 5, 8, and 10.

Name the least common multiple

n. Of 3 and 5.

o. Of 6 and 7.

p. Of 4 and 6.

q. Of 6 and 10.

r. Of 9 and 12.

s. Of 4, 6, and 8.

t. Of 4, 8, and 10.

u. Of 4, 5, and 6.

v. Of 3, 8, and 10.

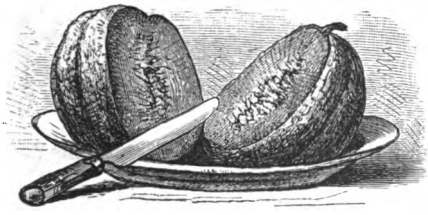
w. Of 6, 9, and 12.

For fuller treatment of Greatest Common Factor and Least Common Multiple, see Appendix, pages 139, 140.

SECTION VIII.

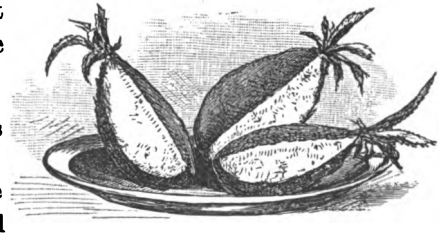
COMMON FRACTIONS.

197. Richard cut a melon into two equal parts and gave his brother one of the parts. What part of the melon did each then have? (Art. **142.**)



a. How many halves equal a whole thing?

198. Three boys divided a pineapple equally among themselves. What part of the pineapple did each boy have?



b. How many thirds equal a whole thing?

c. If any single thing or unit is divided into four equal parts, what is each of the parts called? What are two of the parts called? three of the parts?

d. If a unit is divided into five equal parts, what is each of the parts called? two of the parts? three of the parts? four of the parts?

e. If a unit is divided into six equal parts, what is each of the parts called? two of the parts? three of the parts?

f. In one how many fourths? thirds? fifths? sixths?

199. The equal parts of a unit, as halves, thirds, fifths, etc., are ***fractions.*** Numbers that are not fractions are called ***whole numbers*** or ***integers.***

g. To obtain two thirds, into how many equal parts is the unit divided? How many parts are taken?

200. The number of equal parts into which a unit is divided is the ***denominator*** of the fraction.

Thus, in the fraction two thirds, the denominator is *three.*

201. The number of equal parts taken is the ***numerator*** of the fraction.

Thus, in the fraction two thirds, the numerator is *two.*

202. The numerator and denominator are called the ***terms*** of the fraction.

h. What is the denominator of the fraction three fifths?

i. What is the numerator?

Writing Common Fractions.

203. Two thirds is written as in the margin, the numerator *two* above the line, and the denominator *three* below.

Numerator.....2
Denominator...3

Exercises.

204. Write in figures the following:

a. One half.
b. One third.
c. Two thirds.
d. Three fourths.
e. Four fifths; five fifths.
f. Seven twelfths; seven fifths.

g. Write any fraction you please having for a denominator six; seven; nine; eleven.

h. Write any fraction you please having for a numerator four; six; seven; nine; twelve.

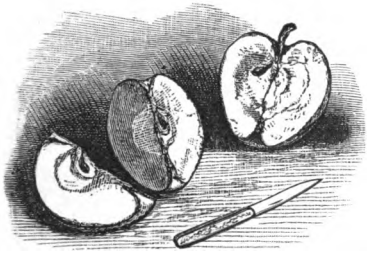
205. Read the following, and tell how many parts the unit is divided into, and how many parts are taken in each case:

i. $\frac{3}{4}$ apple. *k.* $\frac{5}{6}$ day. *m.* $\frac{9}{10}$ \$. *o.* $\frac{6}{8}$. *q.* $\frac{10}{11}$.

j. $\frac{2}{3}$ pear. *l.* $\frac{7}{8}$ quart. *n.* $\frac{4}{5}$ \$. *p.* $\frac{3}{9}$. *r.* $\frac{11}{12}$.

To change a Fraction to smaller or larger Terms.

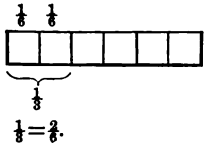
206. Mary cut an apple into halves, and then cut one of the halves into two equal parts. What part of the whole apple was one of these smaller parts?



If Mary should put these two fourths together, what part of the whole apple would they make?

a. One half equals how many fourths?

b. Two fourths equal how many halves?



c. If a piece of paper one inch long be cut into thirds, and each of these thirds be cut into two equal parts, how many equal parts will the whole piece be cut into?

d. What part of an inch will one of these smaller parts be?

e. One third equals how many sixths?

f. Two sixths equal how many thirds?

207. Compare the terms of the equal fractions $\frac{1}{3}$ and $\frac{2}{6}$.

The terms of the fraction $\frac{1}{3}$ are one half as large as the terms of the fraction $\frac{2}{6}$. And the terms of the fraction $\frac{2}{6}$ are twice as large as the terms of the fraction $\frac{1}{3}$. So,

208. *If both terms of a fraction be multiplied by the same number, the value of the fraction will not be changed.* And,

If both terms of a fraction be divided by the same number, the value of the fraction will not be changed.

g. By what will you multiply the terms of the fraction $\frac{1}{2}$ to change it to fourths? to sixths? to eighths?

h. By what will you divide the terms of the fraction $\frac{2}{4}$ to change it to $\frac{1}{2}$?

i. By what will you divide the terms of the following fractions to change them to halves: $\frac{3}{6}$? $\frac{5}{10}$? $\frac{6}{12}$? $\frac{8}{16}$?

209. Oral Exercises.

First do the following examples, writing the answers upon the slate; then practise doing them mentally till you can name the results at sight.

a. Change to smaller terms: $\frac{2}{4}$; $\frac{2}{8}$; $\frac{4}{8}$; $\frac{6}{8}$; $\frac{3}{6}$; $\frac{6}{9}$; $\frac{6}{10}$; $\frac{4}{16}$; $\frac{6}{12}$; $\frac{5}{10}$; $\frac{2}{10}$; $\frac{3}{12}$; $\frac{2}{12}$; $\frac{7}{14}$.

b. Change to smaller terms: $\frac{8}{16}$; $\frac{8}{12}$; $\frac{9}{12}$; $\frac{5}{15}$; $\frac{10}{15}$; $\frac{8}{20}$; $\frac{10}{25}$; $\frac{5}{25}$; $\frac{7}{21}$; $\frac{6}{18}$; $\frac{4}{6}$; $\frac{4}{16}$; $\frac{7}{28}$.

c. Change to smaller terms: $\frac{12}{18}$; $\frac{10}{12}$; $\frac{14}{16}$; $\frac{8}{10}$; $\frac{10}{18}$; $\frac{20}{25}$; $\frac{20}{30}$; $\frac{18}{20}$; $\frac{18}{24}$, $\frac{18}{30}$; $\frac{12}{15}$; $\frac{12}{16}$; $\frac{12}{20}$.

d. Change $\frac{1}{2}$ to fourths; to sixths; to tenths; to twelfths.

e. Change $\frac{1}{3}$ to sixths; to ninths; to twelfths; to fifteenths.

f. Change $\frac{2}{3}$ to sixths; to ninths; to twelfths; to fifteenths.

g. Change to twenty-fourths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{4}$; $\frac{3}{4}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{1}{8}$; $\frac{3}{8}$; $\frac{5}{8}$; $\frac{7}{8}$; $\frac{1}{12}$; $\frac{3}{12}$; $\frac{5}{12}$.

h. Change to thirtieths: $\frac{1}{2}$; $\frac{1}{3}$; $\frac{2}{3}$; $\frac{1}{5}$; $\frac{3}{5}$; $\frac{1}{6}$; $\frac{5}{6}$; $\frac{4}{6}$; $\frac{1}{10}$; $\frac{3}{10}$; $\frac{7}{10}$; $\frac{9}{10}$; $\frac{1}{15}$; $\frac{2}{15}$; $\frac{4}{15}$.

To change a Fraction to its smallest Terms.

210. To change a fraction to its smallest terms, *Divide by all the factors that are common to numerator and denominator; or divide both terms by their greatest common factor.*

211. Examples for the Slate.

Change the following to equivalent fractions of smallest terms:

a. Do the work mentally, and write the result as it is written in *a*.

$\frac{16}{24} = \frac{2}{3}$.

(1.) $\frac{18}{30}$. (4.) $\frac{21}{24}$. (7.) $\frac{16}{36}$. (10.) $\frac{40}{48}$. (13.) $\frac{30}{75}$.

(2.) $\frac{12}{28}$. (5.) $\frac{20}{36}$. (8.) $\frac{24}{40}$. (11.) $\frac{20}{60}$. (14.) $\frac{48}{80}$.

(3.) $\frac{9}{27}$. (6.) $\frac{12}{24}$. (9.) $\frac{14}{42}$. (12.) $\frac{36}{60}$. (15.) $\frac{48}{120}$.

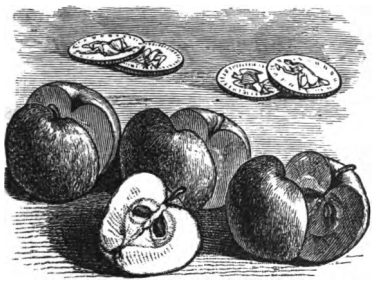
For other examples, see page 129.

To change Improper Fractions to Whole or to Mixed Numbers.

212. A fraction whose numerator equals or exceeds its denominator is called an ***improper fraction.***

Thus, $\frac{4}{4}$, $\frac{5}{4}$, and $\frac{7}{2}$ are called improper fractions.

a. 4 half dollars equal how many dollars? ***b.*** $\frac{7}{2}$ apples equal how many whole apples?



(***b.***) 2) 7

$3\frac{1}{2}$

Since 2 halves make a whole one, $\frac{7}{2}$ apples will equal as many whole apples as there are 2's in 7, which is 3, and 1 half remains. *Ans.* $3\frac{1}{2}$ apples.

213. The number $3\frac{1}{2}$ consists of a whole number and a fraction. A number consisting of a whole number and a fraction is a ***mixed number.***

214. Oral Exercises.

c. Alvin picked berries into a dish that held $\frac{1}{3}$ of a quart; if he filled the dish 8 times, how many quarts did he pick?

d. How many quarts are there in $\frac{8}{3}$ quarts? in $\frac{8}{2}$? in $\frac{11}{3}$?

e. How many dollars are there in \$$\frac{10}{4}$? in \$$\frac{11}{4}$? in \$$\frac{15}{4}$?

f. By what do you divide to change halves to units? to change thirds? fourths? fifths? sevenths? tenths?

g. Change to whole numbers: $\frac{6}{2}$; $\frac{8}{2}$; $\frac{10}{2}$; $\frac{8}{8}$; $\frac{12}{3}$; $\frac{8}{3}$; $\frac{18}{3}$.

h. Change to mixed numbers: $\frac{7}{2}$; $\frac{9}{2}$; $\frac{13}{2}$; $\frac{5}{3}$; $\frac{7}{3}$; $\frac{16}{3}$; $\frac{13}{4}$; $\frac{15}{4}$; $\frac{7}{5}$; $\frac{8}{5}$; $\frac{6}{5}$; $\frac{16}{6}$; $\frac{17}{6}$; $\frac{15}{7}$; $\frac{20}{8}$; $\frac{24}{9}$; $\frac{42}{10}$.

i. Change to whole or mixed numbers: $\frac{8}{2}$; $\frac{12}{2}$; $\frac{8}{3}$; $\frac{11}{3}$; $\frac{12}{4}$; $\frac{24}{4}$; $\frac{16}{5}$; $\frac{19}{5}$; $\frac{18}{6}$; $\frac{42}{6}$; $\frac{19}{7}$; $\frac{28}{7}$; $\frac{24}{8}$; $\frac{37}{8}$; $\frac{16}{9}$; $\frac{63}{9}$; $\frac{48}{10}$.

215. Examples for the Slate.

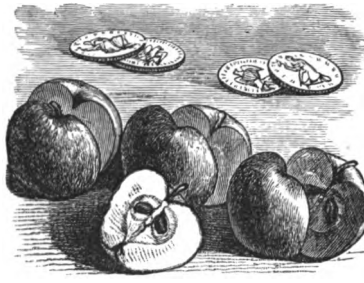
Change the following fractions to whole or mixed numbers, writing the results as in ***a*** and ***b,*** below.

a. $\frac{83}{5} = 16\frac{3}{5}$. (16.) $\frac{181}{6}$. (18.) $\frac{121}{4}$. (20.) $\frac{232}{8}$. (22.) $\frac{410}{11}$.

b. $\frac{49}{3} = 19\frac{2}{3}$. (17.) $\frac{212}{7}$. (19.) $\frac{268}{8}$. (21.) $\frac{561}{9}$. (23.) $\frac{721}{12}$.

To change a Whole or a Mixed Number to an Improper Fraction.

216. *a.* Two dollars equal how many half dollars?



b. How many half apples are there in 3 apples? in 4? in 5? in 7?

c. By what do you multiply to change units to halves?

d. How many half apples are there in $3\frac{1}{2}$ apples?

(*d.*) $3\frac{1}{2} = \frac{7}{2}$.

Since in 1 there are 2 halves, in 3 there are 3 times 2 halves, or 6 halves, which with 1 half added are 7 halves. *Ans.* $\frac{7}{2}$ apples.

217. Oral Exercises.

e. Ada had 2 yards of ribbon, which she made into knots of 1 third of a yard each. How many knots did she make?

f. How many thirds are there in 2? in 3? in 5? in 7?

g. By what do you multiply to change a number to thirds?

h. How many thirds are there in $4\frac{2}{3}$? in $5\frac{1}{3}$? in $6\frac{2}{3}$?

i. If it takes 1 fourth of a yard of cloth to make a cap, how many caps can be made from 5 yards? from $5\frac{3}{4}$ yards?

j. By what do you multiply to change a number to fourths? fifths? sixths? eighths? ninths? tenths?

k. Change 3 to fourths; 4 to fifths; 5 to eighths; 8 to tenths.

l. Change to improper fractions: $1\frac{1}{2}$; $8\frac{1}{2}$; $2\frac{1}{3}$; $4\frac{1}{2}$; $7\frac{1}{4}$; $3\frac{1}{8}$.

m. Change to improper fractions: $1\frac{5}{8}$; $4\frac{1}{3}$; $8\frac{2}{9}$; $3\frac{4}{7}$; $3\frac{7}{10}$; $8\frac{3}{5}$; $6\frac{2}{3}$; $8\frac{3}{4}$; $5\frac{4}{5}$; $7\frac{1}{3}$; $12\frac{1}{2}$; $6\frac{1}{4}$; $5\frac{2}{5}$; $2\frac{1}{9}$; $4\frac{6}{7}$; $3\frac{5}{8}$; $5\frac{2}{3}$; $7\frac{5}{6}$.

218. Examples for the Slate.

Change the following numbers to improper fractions, writing the result as in *a*, below.

a. $16\frac{2}{3} = \frac{50}{3}$.	(26.) $33\frac{1}{3}$.	(29.) $32\frac{2}{5}$.	(32.) $121\frac{1}{3}$.
(24.) $37\frac{1}{3}$.	(27.) $66\frac{2}{3}$.	(30.) $272\frac{1}{4}$.	(33.) $271\frac{1}{7}$.
(25.) $17\frac{1}{4}$.	(28.) $30\frac{1}{4}$.	(31.) $187\frac{5}{8}$.	(34.) $368\frac{7}{8}$.

ADDITION OF FRACTIONS.

To add Fractions having a Common Denominator.

219. *a.* In $\frac{1}{4}$ of a day, $\frac{2}{4}$ of a day, and $\frac{3}{4}$ of a day, how many fourths? How many days?

The fractions $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$ have the same denominator, 4. Fractions which have the same denominator are said to have a *common denominator.*

b. In adding $\frac{1}{4}$, $\frac{2}{4}$, and $\frac{3}{4}$, which terms did you add?

220. Oral Examples.

c. Charles picked $\frac{1}{8}$ of a peck of berries, William picked $\frac{3}{8}$ of a peck, and Alfred picked $\frac{4}{8}$ of a peck. How many eighths did all pick? How many pecks?

d. How many sevenths are $\frac{2}{7}$, $\frac{3}{7}$, and $\frac{1}{7}$?

e. How many sixths are $\frac{1}{6}$, $\frac{2}{6}$, and $\frac{3}{6}$?

f. How many elevenths are $\frac{3}{11}$, $\frac{2}{11}$, and $\frac{5}{11}$?

g. Add $\frac{4}{12}$, $\frac{7}{12}$, and $\frac{11}{12}$.

h. Add $\frac{5}{20}$, $\frac{12}{20}$, and $\frac{11}{20}$.

i. Add $\frac{4}{25}$, $\frac{8}{25}$, and $\frac{7}{25}$.

j. Add $\frac{9}{35}$, $\frac{8}{35}$, and $\frac{6}{35}$.

To add Fractions not having a Common Denominator.

221. *a.* James worked $\frac{3}{4}$ of a day for Mr. Smith and $\frac{1}{2}$ of a day for Mr. Clark. How many fourths of a day did he work for both? How many days?

b. How many eighths in $\frac{1}{2}$ plus $\frac{1}{8}$? in $\frac{1}{4}$ plus $\frac{3}{8}$?

c. In $\frac{2}{3}$ plus $\frac{2}{9}$ how many ninths? How many units?

d. In $\frac{4}{5}$ plus $\frac{7}{10}$ how many tenths? How many units?

222. *e.* Add $\frac{1}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$.

WRITTEN WORK.

$\frac{1}{3} = \frac{4}{12}$
$\frac{3}{4} = \frac{9}{12}$
$\frac{5}{6} = \frac{10}{12}$
$\frac{23}{12} = 1\frac{11}{12}$.

To add these fractions, they must be changed to equivalent fractions having a common denominator.

The most convenient denominator is the least common multiple of all the denominators.

The least common multiple of 3 and 4 is 3 times 4, or 12, which is also a multiple of 6.*

To change $\frac{1}{3}$ to twelfths, the denominator 3 must be multiplied by 4. Hence the numerator must be multiplied by 4. Thus, $\frac{1}{3}$ is found to equal $\frac{4}{12}$.

In a similar way $\frac{3}{4}$ is found to equal $\frac{9}{12}$, and $\frac{5}{6}$ to equal $\frac{10}{12}$. Adding these fractions, we have $\frac{23}{12}$, or $1\frac{11}{12}$, for the sum. *Ans.* $1\frac{11}{12}$.

223. Oral Examples.

f. Add $\frac{1}{2}$ and $\frac{1}{4}$; $\frac{1}{3}$ and $\frac{1}{6}$; $\frac{1}{2}$ and $\frac{1}{6}$; $\frac{1}{4}$ and $\frac{1}{8}$; $\frac{1}{5}$ and $\frac{1}{10}$; $\frac{1}{3}$ and $\frac{1}{9}$; $\frac{1}{3}$ and $\frac{1}{12}$; $\frac{1}{4}$ and $\frac{1}{12}$; $\frac{1}{7}$ and $\frac{1}{14}$.

g. Add $\frac{1}{2}$ and $\frac{1}{3}$; $\frac{1}{4}$ and $\frac{1}{5}$; $\frac{1}{3}$ and $\frac{1}{4}$; $\frac{1}{6}$ and $\frac{1}{4}$; $\frac{1}{6}$ and $\frac{1}{5}$.

h. Add $\frac{1}{2}$ and $\frac{2}{3}$; $\frac{3}{4}$ and $\frac{2}{5}$; $\frac{2}{3}$ and $\frac{3}{4}$; $\frac{5}{6}$ and $\frac{3}{4}$; $\frac{5}{6}$ and $\frac{4}{5}$.

i. Add $\frac{1}{3}$, $\frac{1}{4}$, and $\frac{1}{6}$; $\frac{1}{2}$, $\frac{1}{5}$, and $\frac{1}{4}$; $\frac{1}{6}$, $\frac{1}{5}$, and $\frac{1}{10}$; $\frac{1}{3}$, $\frac{1}{8}$, and $\frac{1}{12}$; $\frac{1}{5}$, $\frac{1}{8}$, and $\frac{1}{10}$.

j. Add $\frac{2}{3}$, $\frac{3}{4}$, and $\frac{5}{6}$; $\frac{1}{2}$, $\frac{2}{5}$, and $\frac{3}{4}$; $\frac{5}{6}$, $\frac{2}{5}$, and $\frac{3}{10}$; $\frac{2}{3}$, $\frac{3}{8}$, and $\frac{5}{12}$; $\frac{2}{5}$, $\frac{3}{8}$, and $\frac{7}{10}$.

k. Miss French cut from a piece of ribbon $\frac{2}{3}$ and $\frac{3}{4}$ of a yard, and then had $\frac{3}{4}$ of a yard left. How long was the piece?

224. Examples for the Slate.

In doing the following examples, write the work as in *e*, Art. 222.

35. Add $\frac{5}{8}$, $\frac{7}{12}$, and $\frac{5}{48}$.
36. Add $\frac{7}{10}$, $\frac{4}{15}$, and $\frac{5}{12}$.
37. Add $\frac{7}{15}$, $\frac{3}{9}$,† and $\frac{2}{3}$.
38. Add $\frac{5}{30}$, $\frac{5}{8}$, and $\frac{1}{7}$.
39. Add $\frac{16}{20}$, $\frac{5}{9}$, and $\frac{5}{18}$.
40. Add $\frac{4}{12}$, $\frac{4}{20}$, and $\frac{4}{7}$.
41. Add $\frac{8}{12}$, $\frac{16}{20}$, and $\frac{4}{25}$.
42. Add $\frac{5}{11}$, $\frac{3}{4}$, and $\frac{7}{22}$.

In doing the following examples, add the whole numbers and the fractions separately.

43. Add $5\frac{1}{3}$, $4\frac{2}{5}$, and $7\frac{1}{2}$.
44. Add $4\frac{7}{8}$, $9\frac{2}{3}$, and $18\frac{4}{5}$.

* If in any example the least common multiple is not readily seen, the pupil may take for a common denominator any common multiple of the denominators. The product of all the denominators is frequently the most convenient.

† What should first be done to this fraction?

SUBTRACTION OF FRACTIONS.

To subtract Fractions having a Common Denominator.

225. Oral Examples.

a. John owned $\frac{7}{8}$ of a football, and sold $\frac{3}{8}$ to Burt. How many eighths did he own then?

b. $\frac{7}{8} - \frac{3}{8} =$ what?

c. $\frac{4}{9} - \frac{2}{9} =$ what?

d. From $\frac{7}{12}$ take $\frac{3}{12}$.

e. From $1\frac{4}{15}$ take $\frac{9}{15}$.

How do you subtract when the minuend and subtrahend have a common denominator?

f. Charles had a melon, and gave $\frac{2}{3}$ of it away. What part had he left?

g. $1 - \frac{2}{3}$? $1 - \frac{7}{8}$? $1 - \frac{9}{10}$? $1 - \frac{3}{11}$? $1 - \frac{7}{9}$? $2 - \frac{7}{9}$?

h. $4 - 2 - \frac{2}{5}$? $4 - 2\frac{2}{5}$? $12 - 3\frac{7}{10}$? $20 - 6\frac{2}{3}$? $100 - 12\frac{1}{2}$?

To subtract Fractions not having a Common Denominator.

226. Oral Examples.

i. Silas could have the use of a boat for half an hour. After he had used it a quarter of an hour, how much longer could he use it?

j. If $\frac{1}{8}$ of a yard of muslin be cut from $\frac{3}{4}$ of a yard, how many eighths will be left?

k. $\frac{1}{2}$ less $\frac{1}{10}$ are how many tenths? How many fifths?

l. $\frac{1}{3}$ less $\frac{1}{9}$ are how many ninths?

227. *m.* From $\frac{3}{4}$ take $\frac{2}{3}$.

WRITTEN WORK.

$\frac{3}{4} = \frac{9}{12}$

$\frac{2}{3} = \frac{8}{12}$

$\frac{1}{12}$

That the subtraction may be performed, these fractions must be changed to equivalent fractions having a common denominator. The least common denominator is 12. $\frac{3}{4} = \frac{9}{12}$, and $\frac{2}{3} = \frac{8}{12}$. $\frac{9}{12} - \frac{8}{12} = \frac{1}{12}$. *Ans.* $\frac{1}{12}$.

n. $\frac{1}{3} - \frac{1}{4}$? $\frac{1}{4} - \frac{1}{5}$? $\frac{1}{5} - \frac{1}{6}$? $\frac{1}{7} - \frac{1}{8}$? $\frac{1}{8} - \frac{1}{9}$?

o. $\frac{2}{3} - \frac{1}{4}$? $\frac{3}{4} - \frac{2}{5}$? $\frac{3}{5} - \frac{2}{6}$? $\frac{4}{7} - \frac{3}{8}$? $\frac{5}{8} - \frac{4}{9}$?

228. Examples for the Slate.

In the following examples, write the work as in *m*, Art. 227.

(45.) $\frac{5}{8} - \frac{6}{12}$ = what?

(46.) $\frac{4}{5} - \frac{3}{7}$ = what?

(47.) $\frac{8}{15} - \frac{1}{12}$ = what?

(48.) $\frac{9}{21} - \frac{2}{5}$ = what?

(49.) $17 - 2\frac{5}{8}$ = what?

(50.) $1\frac{7}{24} - \frac{2}{5}$ = what?

(51.) $1\frac{1}{2} - \frac{7}{9}$ = what?

(52.) $12\frac{1}{3} - 4\frac{5}{8}$ = what?

229. Examples in Addition and Subtraction.

53. What are the contents of three remnants of carpeting containing as follows: $24\frac{1}{8}$ yards, $8\frac{1}{3}$ yards, and $18\frac{7}{9}$ yards?

54. Clifford had $2\frac{3}{4}$ miles to row. After rowing $1\frac{3}{4}$ miles, how much farther had he to row?

55. A man sold $4\frac{1}{2}$ bushels of cranberries at one time, $3\frac{3}{4}$ bushels at another, and $7\frac{7}{12}$ at another. How many bushels did he sell in all?

56. If it took $12\frac{2}{3}$ hours to plough a field, and $60\frac{1}{8}$ hours to plant it, how many more hours did it take to plant than to plough it?

57. A farm of 23 acres contained a peach-orchard of $4\frac{1}{8}$ acres, a wheat-field of $5\frac{1}{2}$ acres, a corn-field of $8\frac{1}{4}$ acres; $\frac{3}{4}$ of an acre was planted with peas, $1\frac{1}{3}$ acres with strawberries, and the rest was a garden. How much was a garden?

58. How many more acres of the above farm were there in the corn-field than in the wheat-field?

59. In making some boxes a carpenter used $4\frac{1}{4}$ pounds of board-nails, $1\frac{1}{2}$ pounds of shingle-nails, and $6\frac{5}{8}$ pounds of lath-nails. How many pounds did he use in all?

60. From a chest of tea containing $42\frac{5}{8}$ pounds, $17\frac{9}{16}$ pounds were sold. How many pounds were left?

61. A boy once paid his fare of $\$2\frac{1}{10}$ from Springfield to Chatham, but, falling asleep, was carried on to Albany, where he paid $\$1\frac{1}{4}$ for his lodging and breakfast, and $\$\frac{3}{10}$ to return to Chatham. What was the entire expense of his trip?

For other examples in Addition and Subtraction, see page 129.

MULTIPLICATION.

To multiply a Fraction by an Integer.

230. If James earns $\frac{2}{3}$ of a dollar in 1 day, how much will he earn in 2 days?

Solution. — If he earns $\frac{2}{3}$ of a dollar in 1 day, in 2 days he will earn 2 times $\frac{2}{3}$ of a dollar, which equals $\frac{4}{3}$, or $1\frac{1}{3}$ dollars. *Ans.* $1\frac{1}{3}$ dollars.

In multiplying $\frac{2}{3}$ by 2, which term of the fraction is multiplied?

231. Oral Examples.

a. If it takes Dana $\frac{1}{4}$ of an hour to walk a mile, how long will it take him to walk 2 miles? 3 miles? 6 miles? 8 miles?

b. How many dollars will it take to give $\frac{1}{2}$ of a dollar to each of 4 boys? 5 boys? 8 boys? 9 boys?

c. How many are 4 times $\frac{1}{4}$? 7 times $\frac{1}{4}$? 7 times $\frac{1}{5}$? 7 times $\frac{2}{5}$? 7 times $\frac{3}{4}$? 5 times $\frac{2}{7}$? 4 times $\frac{3}{5}$?

d. How many are 6 times $\frac{3}{8}$? 8 times $\frac{5}{6}$? 9 times $1\frac{1}{3}$?

NOTE. To multiply $1\frac{1}{3}$, multiply the integer and the fraction separately.

e. Multiply $4\frac{5}{6}$ by 6; $7\frac{2}{3}$ by 8; $8\frac{2}{7}$ by 9; $9\frac{1}{3}$ by 8.

232. Examples for the Slate.

f. Multiply $\frac{3}{5}$ by 25.

WRITTEN WORK.

$$\frac{3 \times 25}{5} = \frac{75}{5} = 15.$$

or

$$\frac{3 \times \overset{5}{\cancel{25}}}{\underset{1}{\cancel{5}}} = 15.$$

In doing the following examples, write the work as it is written in Example *f*. If there are common factors in the numbers written above and below the line, strike them out as is done in the second form of the written work.

233. Striking out equal factors in the dividend and divisor is ***cancelling***.

62. Multiply $\frac{5}{8}$ by 16.
63. Multiply $\frac{4}{9}$ by 63.
64. Multiply $\frac{7}{10}$ by 15.
65. Multiply $1\frac{1}{3}$ by 14.

234. *g.* Multiply $16\frac{1}{2}$ by 11.

WRITTEN WORK.

$16\frac{1}{2}$
11
$5\frac{1}{2}$
176
$181\frac{1}{2}$

Multiplying $\frac{1}{2}$ by 11, we have $\frac{11}{2}$, which equals $5\frac{1}{2}$. Multiplying 16 by 11, we have 176, which, with $5\frac{1}{2}$, gives $181\frac{1}{2}$. *Ans.* $181\frac{1}{2}$.

66. Multiply $5\frac{3}{4}$ by 7. 68. Multiply $16\frac{2}{3}$ by 10.

67. Multiply $12\frac{3}{8}$ by 12. 69. Multiply $66\frac{2}{3}$ by 25.

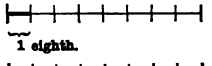
70. Simon can walk a mile in $14\frac{1}{2}$ minutes. How long will it take him to walk 3 miles?

71. If a lot of cloth can be woven in $12\frac{3}{10}$ days on 4 looms, in how many days can it be woven on 1 loom?

72. Eight persons bought a tent together, each paying as his share $\$15\frac{5}{8}$. What was the price of the tent?

To multiply an Integer by a Fraction.

235. What is $\frac{1}{8}$ of 2 yards?



If $\frac{1}{8}$ of each of 2 yards be taken, we shall have $\frac{2}{8}$ of a yard. *Ans.* $\frac{2}{8}$ of a yard. (See illustration; also page 58, Example *o*.)

$\frac{1}{8}$ of $2 = \frac{2}{8}$.

236. Oral Examples.

a. What is $\frac{1}{3}$ of 2? $\frac{1}{4}$ of 3? $\frac{1}{5}$ of 3? $\frac{1}{10}$ of $\$8$? $\frac{1}{5}$ of 8 days?

b. If 1 yard of tape is worth 3 cents, what is $\frac{2}{5}$ of a yard worth?

Solution. — If 1 yard is worth 3 cents, $\frac{2}{5}$ of a yard is worth $\frac{2}{5}$ of 3 cents. $\frac{1}{5}$ of 3 is $\frac{3}{5}$, and $\frac{2}{5}$ of 3 is 2 times $\frac{3}{5}$, which is $\frac{6}{5}$, or $1\frac{1}{5}$. *Ans.* $1\frac{1}{5}$ cents.

c. What is $\frac{1}{3}$ of 4? $\frac{2}{3}$ of 4? $\frac{2}{3}$ of 14? $\frac{2}{3}$ of 17? $\frac{2}{3}$ of 19?

d. What is $\frac{1}{4}$ of 7? $\frac{3}{4}$ of 7? $\frac{1}{8}$ of 13? $\frac{3}{8}$ of 13? $\frac{3}{8}$ of 17?

237. Finding the fractional part of a number is called ***multiplying by a fraction.***

Thus, in finding $\frac{3}{8}$ of 17 we are said to multiply 17 by $\frac{3}{8}$.

238. Examples for the Slate.

e. Multiply 17 by $\frac{3}{5}$. *f.* Multiply 24 by $1\frac{5}{8}$.

WRITTEN WORK.

(*e.*) $\frac{17 \times 3}{5} = \frac{51}{5} = 10\frac{1}{5}$. *Ans.*

(*f.*) $\frac{\overset{4}{\cancel{24}} \times 5}{\underset{3}{\cancel{15}}} = \frac{20}{3} = 6\frac{2}{3}$. *Ans.*

To multiply 17 by $\frac{3}{5}$ is to take $\frac{3}{5}$ of 17.

To find $\frac{3}{5}$ of 17 we may either divide by 5 and then multiply the result by 3, or we may first multiply by 3 and then divide the result by 5. The latter method is generally easier, as in Examples *e* and *f*.

73. What is $\frac{5}{6}$ of 82?
74. What is $\frac{6}{7}$ of 324?
75. What is $\frac{4}{11}$ of 581?
76. Multiply 18 by $1\frac{5}{12}$.
77. Multiply 30 by $\frac{7}{8}$.
78. Multiply 35 by $2\frac{4}{11}$.

79. If a barrel of flour costs $8.75, what will $\frac{3}{5}$ of a barrel cost?

80. If a wheel turns 480 times in going a mile, how many times will it turn in going $\frac{5}{8}$ of a mile?

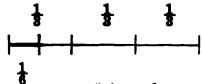
81. If a clerk can copy 50 pages in a day, how many pages can he copy in $2\frac{1}{2}$ days? in $3\frac{1}{3}$ days?

82. What cost $14\frac{1}{2}$ yards of cambric at 42 cents a yard?

83. What cost $16\frac{3}{4}$ yards of silk at $2.75 a yard?

To multiply a Fraction by a Fraction.

239. What is $\frac{1}{2}$ of $\frac{1}{3}$?



To find $\frac{1}{2}$ of $\frac{1}{3}$, the $\frac{1}{3}$ must be divided into 2 equal parts.

Since the entire unit will contain 3 times 2 or 6 such parts, one of the parts will be $\frac{1}{6}$ of the unit. *Ans.* $\frac{1}{6}$.

In finding $\frac{1}{2}$ of $\frac{1}{3}$, how was the new denominator obtained?

240. Oral Examples.

a. What is $\frac{1}{2}$ of $\frac{1}{4}$? $\frac{1}{2}$ of $\frac{1}{5}$? $\frac{1}{2}$ of $\frac{1}{6}$? $\frac{1}{3}$ of $\frac{1}{2}$? $\frac{1}{3}$ of $\frac{1}{4}$?

b. What is $\frac{1}{3}$ of $\frac{1}{6}$? $\frac{1}{4}$ of $\frac{1}{5}$? $\frac{1}{5}$ of $\frac{1}{3}$? $\frac{1}{5}$ of $\frac{1}{6}$? $\frac{1}{6}$ of $\frac{1}{2}$?

241. *c.* What is $\frac{2}{3}$ of $\frac{4}{5}$?

WRITTEN WORK. $\frac{4}{5} \times \frac{2}{3} = \frac{8}{15}$.

$\frac{1}{5}$ of $\frac{1}{5}$ is $\frac{1}{15}$, $(\frac{1}{5\times3})$; then $\frac{1}{3}$ of $\frac{4}{5}$ must be $\frac{4}{15}$, $(\frac{4}{5\times3})$, and $\frac{2}{3}$ of $\frac{4}{5}$ must be 2 times as much, or $\frac{8}{15}$, $(\frac{4\times2}{5\times3})$. *Ans.* $\frac{8}{15}$.

In finding $\frac{2}{3}$ of $\frac{4}{5}$, how was the new numerator obtained? the new denominator?

To multiply a fraction by a fraction, *Find the product of the numerators for the new numerator, and the product of the denominators for the new denominator.*

d. Find $\frac{2}{3}$ of $\frac{1}{4}$; $\frac{1}{4}$ of $\frac{2}{3}$; $\frac{1}{3}$ of $\frac{2}{5}$; $\frac{2}{5}$ of $\frac{1}{3}$; $\frac{2}{5}$ of $\frac{3}{4}$.

e. Find $\frac{1}{3}$ of $\frac{5}{6}$; $\frac{5}{6}$ of $\frac{2}{3}$; $\frac{4}{5}$ of $\frac{2}{7}$; $\frac{1}{7}$ of $\frac{2}{3}$; $\frac{3}{5}$ of $\frac{3}{8}$.

f. If a pound of butter costs \$ $\frac{3}{10}$, what part of a dollar will $\frac{1}{4}$ of a pound cost? What will $\frac{3}{4}$ of a pound cost?

g. A paper-hanger had $\frac{3}{4}$ of a barrel of flour, and used $\frac{2}{3}$ of it for paste. What part of the barrel did he use?

242. Examples for the Slate.

84. What is $\frac{2}{5}$ of $\frac{5}{12}$?
85. What is $\frac{5}{8}$ of $\frac{7}{10}$?
86. What is $\frac{4}{5}$ of $\frac{8}{11}$?
87. What is $\frac{5}{9}$ of $\frac{4}{25}$?

In the following examples change all mixed numbers to improper fractions before multiplying.

88. Find $\frac{2}{3}$ of $\frac{7}{2}$.
89. Find $\frac{5}{8}$ of $14\frac{2}{7}$.
90. Find $\frac{6}{7}$ of $18\frac{1}{3}$.
91. Find $\frac{7}{12}$ of $3\frac{1}{7}$.
(92.) $3\frac{4}{5} \times 5\frac{1}{2}$?
(93.) $6\frac{1}{8} \times 4\frac{2}{3}$?
94. What cost $3\frac{1}{2}$ barrels of peas at \$ $6\frac{2}{3}$ a barrel?
95. Dora is $15\frac{5}{8}$ years old, and Oscar is $\frac{3}{4}$ as old as Dora. How old is Oscar?

243. Repeat the following table:

Long Measure.

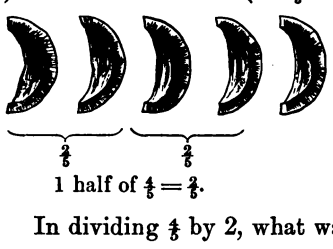
12 inches = 1 foot. $5\frac{1}{2}$ yards or $16\frac{1}{2}$ feet = 1 rod.
3 feet = 1 yard. 320 rods or 5280 feet = 1 mile.

96. How many inches are there in $16\frac{1}{2}$ feet?
97. How many feet are there in $33\frac{1}{3}$ yards?
98. Change 160 rods to yards.
99. Change $3\frac{1}{3}$ rods to feet.

For other examples in Multiplication of Fractions, see page 129.

DIVISION.

To divide a Fraction by an Integer.



1 half of $\frac{4}{5} = \frac{2}{5}$.

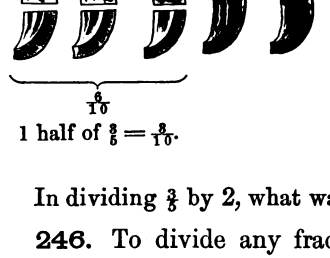
244. If $\frac{4}{5}$ of a melon be divided equally between 2 boys, what part of the melon will each boy have?

If $\frac{4}{5}$ of a melon be divided equally between 2 boys, each boy will have 1 half of $\frac{4}{5}$, or $\frac{2}{5}$ of a melon.

Ans. $\frac{2}{5}$ of a melon.

In dividing $\frac{4}{5}$ by 2, what was done to the numerator?

245. If $\frac{3}{5}$ of a melon be divided equally between two boys, what part of the melon will each boy have?



1 half of $\frac{3}{5} = \frac{3}{10}$.

As the number of parts, 3, cannot be divided by 2 without a remainder, each one of the parts, fifths, may be divided into two equal parts; these parts will be tenths of the whole melon. There will then be 6 tenths, of which each boy's part will be 3 tenths.

In dividing $\frac{3}{5}$ by 2, what was done to the denominator?

246. To divide any fraction by an integer, *Divide the numerator or multiply the denominator by the integer.*

247. Oral Examples.

a. Divide $\frac{2}{3}$ by 2; $\frac{4}{5}$ by 2; $\frac{6}{7}$ by 3; $\frac{8}{9}$ by 4; $\frac{10}{11}$ by 5.

b. Divide $\frac{2}{3}$ by 3; $\frac{3}{5}$ by 2; $\frac{4}{7}$ by 3; $\frac{7}{8}$ by 4; $\frac{9}{10}$ by 5.

c. If 2 girls divide $\frac{2}{3}$ of a yard of ribbon equally between themselves, what part of a yard will each have?

d. If 9 shrubs cost \$ $2\frac{1}{4}$, what is the cost of each?

(*d.*) $2\frac{1}{4} = \frac{9}{4}$. $\frac{\overset{1}{\cancel{9}}}{4 \times \underset{1}{\cancel{9}}} = \frac{1}{4}$. First change the mixed number to an improper fraction.

e. If \$5 was paid for $2\frac{1}{2}$ feet of wood, what was the cost of one foot?

f. In a boarding-house 7 breakfast tickets were sold for $\$1\frac{1}{2}$, 7 dinner tickets for $\$1\frac{3}{4}$, and 7 supper tickets for $\$1\frac{1}{4}$. What was the cost apiece of each kind?

g. A family put up $8\frac{1}{3}$ bushels of potatoes for the 3 winter months. What was that a month?

(*g.*) 3) $8\frac{1}{3}$ $\frac{1}{3}$ of 8 is 2, with a remainder of 2.
$2\frac{7}{9}$ 2 units equal $\frac{6}{3}$, which, with $\frac{1}{3}$, are $\frac{7}{3}$. $\frac{1}{3}$ of $\frac{7}{3}$ is $\frac{7}{9}$.
Ans. $2\frac{7}{9}$ bushels.

h. If the railroad fares of 5 persons are $\$31\frac{1}{4}$, what is the fare for each person?

i. What must be the width of matting of which it takes 5 breadths to reach $8\frac{3}{4}$ yards?

j. What must be the width of paper of which it requires 10 breadths to reach $24\frac{1}{8}$ feet?

k. Divide $1\frac{2}{7}$ by 6; $2\frac{2}{3}$ by 4; $4\frac{1}{4}$ by 3; $8\frac{3}{4}$ by 11.

l. Divide $15\frac{1}{3}$ by 4; by 5; by 6; by 7; by 8; by 9.

248. Examples for the Slate.

100. Mrs. Grant gave $\$10\frac{1}{8}$ for 27 Christmas wreaths. How much did she give apiece?

101. A gentleman paid $\$67\frac{1}{2}$ for board for 120 days. What was the price per day?

102. If 1 needle-woman can do a piece of work in $11\frac{5}{8}$ days, in what time can 12 do it? In what time can 10 do it?

103. If a certain field can be reaped in $15\frac{2}{3}$ days, in what time could a field that is $\frac{1}{10}$ as large be reaped?

104. If $119\frac{1}{4}$ yards of cambric are required to make 9 dresses, how much is required for 1 dress?

105. If a railroad car runs $122\frac{7}{32}$ miles in 5 hours, what is the average rate an hour?

106. Divide $1\frac{7}{12}$ by 25.
107. Divide $\frac{3}{4}\frac{2}{8}$ by 26.
108. Divide $\frac{4}{5}\frac{8}{3}$ by 110.

(109.) $36\frac{3}{8} \div 11$.
(110.) $16\frac{3}{5} \div 20$.
(111.) $272\frac{1}{4} \div 18$.

To divide an Integer or a Fraction by a Fraction.

249. Oral Examples.

a. In one there are how many halves? thirds? fourths?

b. In two there are how many halves? thirds? fourths?

c. Julian had 5 cents, which he spent for pears at $\frac{1}{2}$ cent apiece. How many pears did he buy?

d. How many pears could he buy at $\frac{1}{3}$ of a cent apiece? at $\frac{1}{4}$ of a cent apiece?

e. How many mugs at $\$\frac{1}{10}$ each can you buy for \$3? for \$5?

250. *f.* Helen made 4 knots of worsted into tassels, putting $\frac{2}{3}$ of a knot into each tassel. How many tassels did she make?

WRITTEN WORK.

(*f.*) $4 = \frac{12}{3}$

$\frac{12}{3} \div \frac{2}{3} = 12 \div 2 = 6.$

She must have made as many tassels as there are times $\frac{2}{3}$ in 4.

We first change 4 to thirds, making $\frac{12}{3}$. There are as many times $\frac{2}{3}$ in $\frac{12}{3}$ as there are 2's in 12, or 6. *Ans.* 6 tassels.

NOTE. For a different analysis of this example, see Appendix, page 141.

g. Miss Breck made 6 yards of crash into towels, putting $\frac{3}{4}$ of a yard in each towel. How many did she make?

h. Divide 6 by $\frac{3}{4}$; by $\frac{4}{5}$; by $\frac{1}{2}$; by $\frac{3}{5}$; by $\frac{5}{8}$.

i. Divide 8 by $\frac{1}{4}$; by $\frac{3}{4}$; by $1\frac{1}{4}$ or $\frac{5}{4}$; by $2\frac{1}{3}$ or $\frac{7}{3}$.

j. How many hats at $\$1\frac{1}{2}$ each can be bought for \$12?

How do you change an integer to divide it by a fraction? How do you then divide? How do you divide by a mixed number?

251. *k.* Divide $\frac{8}{9}$ by $\frac{2}{3}$.

l. Divide $\frac{4}{5}$ by $\frac{2}{3}$.

m. Divide $\frac{6}{8}$ by $\frac{3}{8}$.

n. Divide $3\frac{1}{3}$ by $\frac{2}{3}$.

o. $2\frac{2}{3}$ are how many times $\frac{2}{3}$?

p. $4\frac{2}{3}$ are how many times $\frac{2}{3}$?

q. $3\frac{3}{4}$ are how many times $1\frac{1}{4}$?

r. $5\frac{1}{3}$ are how many times $2\frac{2}{3}$?

s. In a farm-school $5\frac{1}{4}$ acres of land were given to some boys to plant, each boy having $\frac{1}{4}$ of an acre. To how many boys was the land given?

When fractions have a common denominator, how do you divide?

252. *t.* How many feet of rope can be made from $1\frac{3}{4}$ pounds of hemp, there being $\frac{1}{6}$ of a pound to a foot?

WRITTEN WORK.

(*t.*) $1\frac{3}{4} = \frac{7}{4} = \frac{21}{12}$; $\frac{1}{6} = \frac{2}{12}$.

$\frac{21}{12} \div \frac{2}{12} = 21 \div 2 = 10\frac{1}{2}$.

As many feet can be made as there are times $\frac{1}{6}$ in $1\frac{3}{4}$ or $\frac{7}{4}$.

$\frac{7}{4}$ and $\frac{1}{6}$ changed to fractions having a common denominator are $\frac{21}{12}$ and $\frac{2}{12}$.

$\frac{21}{12}$ divided by $\frac{2}{12}$ equals 21 divided by 2, or $10\frac{1}{2}$. *Ans.* $10\frac{1}{2}$ feet.

When fractions have different denominators, how do you prepare them to divide? How do you then divide?

253. To divide an integer or a fraction by a fraction, *Change the dividend and divisor to fractions having a common denominator, and then divide the numerator of the dividend by the numerator of the divisor.* (See Appendix page 141.)

u. How many times is $\frac{2}{3}$ contained in $\frac{3}{4}$? $\frac{2}{5}$ in $\frac{3}{5}$? $\frac{3}{5}$ in $\frac{5}{6}$?

v. How many times is $\frac{3}{10}$ contained in $\frac{4}{5}$? $\frac{2}{5}$ in $\frac{4}{6}$? $\frac{5}{6}$ in $1\frac{1}{2}$?

w. I have a dish which holds $\frac{2}{3}$ of a quart. How many times must it be filled to measure $2\frac{1}{2}$ quarts?

x. How many house-lots each containing $\frac{1}{4}$ of an acre can be made from a lot containing $2\frac{1}{2}$ acres?

254. Examples for the Slate.

112. What is $36 \div \frac{9}{10}$?
113. What is $48 \div 1\frac{7}{8}$?
114. What is $52 \div 3\frac{1}{3}$?
115. What is $100 \div 6\frac{1}{4}$?
116. What is $4\frac{1}{2} \div 1\frac{2}{3}$?
117. Divide $\frac{5}{7}$ by $\frac{5}{14}$.
118. Divide $\frac{8}{9}$ by $1\frac{1}{3}$.
119. Divide $33\frac{1}{3}$ by $4\frac{1}{6}$.
120. Divide $1\frac{6}{8}$ by $7\frac{1}{5}$.
121. Divide $8\frac{1}{3}$ by $4\frac{1}{6}$.

122. How many lengths of fencing $\frac{5}{8}$ of a rod long will it take to enclose a square lot of land, each side 40 rods long?

123. The rations of a ship's company were $1\frac{1}{4}$ pounds of meat a day to each person, but after losing a part of their provisions in a storm, they were allowed only $\frac{9}{10}$ as much. What were their rations then?

For other examples in Division, see page 129.

To find the Whole when a Part is given.

Oral Exercises.

255. *a.* If $\frac{1}{2}$ pound of yarn costs 12 cents, what will a pound cost at the same rate?

b. In $\frac{1}{4}$ of a ream of paper there are 5 quires. How many quires are there in a ream?

c. Mr. Fitch sold a boat and gained \$8, which was $\frac{1}{7}$ of what it cost him. How much did it cost him?

d. 8 is $\frac{1}{7}$ of what number?

e. A man sold a wagon and lost \$11, which was $\frac{1}{12}$ of what it cost him. What did it cost him?

f. If $\frac{1}{3}$ of a yard of silk cost \$$1\frac{1}{2}$, what will 1 yard cost?

g. From a cask of oil 15 gallons leaked out. If this was $\frac{1}{10}$ of the contents of the cask, what did the cask contain at first?

h. Frank has a sister whose age equals $\frac{5}{6}$ of his own age. If the difference between their ages is 2 years, what is Frank's age?

Suggestion. Frank's age is $\frac{6}{6}$ of itself. The difference between $\frac{6}{6}$ and $\frac{5}{6}$, which is $\frac{1}{6}$, is 2 years.

i. After losing $\frac{3}{4}$ of his money, a man had \$12 left. How much money had he at first?

j. A man traded off a cow and got $\frac{2}{3}$ of what she cost him. If he gained \$5, what did she cost him?

k. The difference between $\frac{1}{2}$ and $\frac{1}{4}$ of a bushel is 8 quarts. How many quarts are there in a whole bushel?

l. The difference between $\frac{4}{5}$ and $\frac{9}{10}$ of a number is 5. What is that number?

m. The difference between $\frac{1}{2}$ and $\frac{1}{3}$ of a number is $1\frac{1}{2}$. What is that number?

n. \$5 is $\frac{1}{3}$ of how many dollars?

o. $1\frac{1}{2}$ is $\frac{1}{3}$ of what number? $\frac{1}{7}$ of what number?

p. $6\frac{2}{3}$ is $\frac{1}{3}$ of what number? $\frac{1}{8}$ of what number?

256. *a.* If $\frac{3}{5}$ of a piece of work can be performed in 4 days, in how many days can $\frac{1}{5}$ of the work be performed? In how many days can the whole work be performed?

b. If $\frac{3}{5}$ of a number is 4, what is the whole number?

Solution. — If $\frac{3}{5}$ of a number is 4, $\frac{1}{5}$ of the number is $\frac{1}{3}$ of 4, which is 2, and $\frac{5}{5}$ of the number is 5 times 2, or 10. *Ans.* 10.

c. A man performed $\frac{3}{4}$ of a journey in 15 days. In how many days could he perform $\frac{1}{4}$ of the journey? In how many days could he perform the whole journey?

d. If $\frac{3}{4}$ of a number is 15, what is the whole number?

e. If $\frac{5}{6}$ of a quire of paper cost 10 cents, what did the whole quire cost?

f. $\frac{5}{8}$ of a certain number is 10. What is that number?

g. If there are 21 lines in $\frac{7}{9}$ of a page, how many lines are there in the whole page?

h. 21 is $\frac{7}{9}$ of what number?

i. A grocer sold eggs at 20¢ a dozen, which was $\frac{5}{4}$ of what they cost him. What did they cost him?

j. A man gained $15 by selling candy for $\frac{7}{4}$ as much money as it cost him. What did it cost?

k. 15 is $\frac{3}{4}$ of what number?

l. Edgar sold a kite for $\frac{3}{5}$ of what it cost him, and thereby lost 8 cents. What did the kite cost him?

m. 8 is $\frac{2}{3}$ of what number?

n. By selling beef at $6\frac{1}{4}$¢ a pound, a market-man gets but $\frac{5}{8}$ of what it cost him. How much did it cost him?

o. $6\frac{1}{4}$ is $\frac{5}{8}$ of what number?

p. After $\frac{1}{7}$ of a certain month is past there are 24 days remaining. How many days are there in the month?

q. A farmer finds after selling $\frac{2}{7}$ of his pears that he has $8\frac{1}{3}$ barrrels left. How many barrels had he at first?

r. $8\frac{1}{3}$ is $\frac{5}{7}$ of what number?

s. $1\frac{9}{11}$ is $\frac{5}{7}$ of what number?

t. $4\frac{1}{2}$ is $\frac{3}{10}$ of what number?

u. $\frac{1}{2}$ is $\frac{3}{8}$ of what?

v. $\frac{1}{3}$ is $\frac{4}{5}$ of what?

w. $\frac{1}{5}$ is $\frac{2}{7}$ of what?

257. Miscellaneous Oral Examples.

a. At $\$\frac{1}{2}$ a day for board, how many days' board may be obtained for $\$7\frac{1}{2}$?

b. If $\frac{3}{4}$ of a cord of wood costs \$6, what will 1 cord cost?

c. If 5 men can do a piece of work in $\frac{6}{7}$ of a week, in what time can 1 man do it?

d. A man bought a quantity of drugs for \$53, and was obliged to sell them for $\frac{9}{10}$ of what they cost him. How much did he receive for them?

e. A miller paid 90 cents a bushel for wheat, and sold it so as to lose $\frac{2}{9}$ as much as it cost. What did he receive for it?

f. If $\frac{3}{4}$ of a bale of cotton cost \$30, what will 1 bale cost? How many yards of cloth at \$5 a yard will pay for a bale of cotton?

g. If 9 bottles of ink cost $\$2\frac{1}{4}$, what will 5 bottles cost?

h. If 6 pounds of lead cost $\$1\frac{1}{2}$, what will 7 pounds cost?

i. How many packages of $\frac{3}{4}$ of a pound each can be made from 9 pounds of starch?

j. If $\frac{6}{7}$ of a barrel of oil cost \$36, what cost $\frac{5}{8}$ of a barrel?

k. If $\$\frac{3}{4}$ will buy 6 gallons of oil, how much will $\$\frac{1}{2}$ buy?

l. If 20 cents are paid for $\frac{4}{5}$ of a basket of peaches, what is the cost of $\frac{3}{5}$ of the basket?

m. If $\frac{3}{5}$ of a box of raisins is worth \$4, how many clocks at \$2 apiece will pay for a whole box?

n. Mr. Wing sold 6 pigs for \$24, which was $\frac{4}{5}$ of what he paid for them. What did he pay apiece for them?

o. If a man by travelling 8 hours a day can perform a journey in $1\frac{3}{4}$ days, in how many days can he perform it by travelling 10 hours a day?

p. I sold a horse for \$48, which was $\frac{8}{9}$ of what he cost me. When I bought the horse I paid for him with hay at \$10 a ton. How many tons did it take?

q. If a man can walk at the rate of $\frac{3}{8}$ of a mile in $7\frac{1}{2}$ minutes, in what time can he walk 1 mile? 5 miles?

258. Miscellaneous Examples for the Slate.

124. Mr. Sage has 4 pieces of velvet; the first contains $7\frac{3}{4}$ yards, the second $8\frac{3}{4}$ yards, the third $5\frac{7}{16}$ yards, and the fourth $8\frac{1}{2}$ yards. How many yards are there in all?

125. At $\$5\frac{1}{8}$ a ream, what cost $\frac{3}{4}$ of a ream of paper?

126. Mr. Gray bought $18\frac{3}{4}$ acres of land, and gave 1 half of it to his son and 1 half of the remainder to his nephew. How much did he give to his nephew?

127. Emma's dress cost $\$11\frac{1}{4}$, and Mary's $\frac{2}{3}$ as much. What did Mary's cost?

128. Mr. Brown had a building-lot that measured $12\frac{1}{2}$ rods in front. What did it measure in yards? in feet?

129. How many rods of fencing on both sides of a road that is $1\frac{1}{3}$ miles long?

130. What is the cost of fencing a road which is $\frac{3}{4}$ of a mile in length, at $\$14\frac{1}{2}$ a rod?

131. Olive had $\$11\frac{3}{5}$. If she spent all but $\$5\frac{7}{16}$ of it for cambric at $\$\frac{1}{2}$ a yard, how many yards did she buy?

132. Winthrop received $\$18.10$ for work, and after paying a debt of $\$4.80$, had money enough to pay for $3\frac{1}{2}$ weeks' board. What was the price of board per week?

133. Mr. Channing bought land for $\$11\frac{3}{4}$ an acre, and sold it for $\$19\frac{1}{8}$ an acre. What did he gain on 1 acre? on 3 acres?

134. Mr. Howes sold 12 dozen cabbages at $8\frac{1}{3}¢$ apiece, and received in payment 46 pounds of beef at $9¢$ a pound, and the rest in money. What did he receive in money?

135. How many dominos can be cut from a piece of ivory $7\frac{1}{2}$ inches long, allowing $1\frac{1}{4}$ inches for each domino?

136. How many checkers can be made from a block 15 inches long, if $\frac{3}{8}$ of an inch is allowed for each checker?

137. A man owned $23\frac{3}{8}$ acres of meadow, $17\frac{1}{8}$ acres of pasture, and $5\frac{1}{2}$ acres of woodland. How many acres did he own in all?

138. What will $25\frac{1}{2}$ yards of carpeting cost at $\$1\frac{3}{4}$ per yard?

139. At $4\frac{1}{2}$ per week, what is the cost of board for $3\frac{1}{4}$ weeks?

140. How many rations of $1\frac{1}{4}$ pounds each can be furnished from a barrel of beef weighing 200 pounds?

141. Three boys started together to skate down a river. One skated a mile in $8\frac{1}{2}$ minutes, another in $10\frac{1}{3}$ minutes, and the third in $9\frac{1}{8}$ minutes. How much longer did it take the second than the first? than the third?

142. These boys started together to return, and found that it took them $11\frac{1}{2}$ minutes, $12\frac{2}{3}$ minutes, and $13\frac{1}{8}$ minutes respectively, to skate the mile. How many minutes had the first to rest before the second came up? How many before the third came up?

143. When 1 pound of cotton is put into $3\frac{1}{2}$ yards of cloth, how much is put into half a yard? into $12\frac{1}{2}$ yards?

144. If 30 yards of cloth 1 yard wide weigh $12\frac{3}{4}$ pounds, what will 30 yards of the same kind of cloth weigh if it is $\frac{1}{2}$ of a yard wide? if it is $\frac{3}{8}$ of a yard wide?

145. After going $\frac{1}{8}$ of his journey, Mr. Otis had $32\frac{1}{2}$ miles to go. What was the length of his journey?

146. Homer's saddle cost $25, which was $\frac{2}{10}$ of what his horse cost. What did his horse cost?

147. A man sold a watch for $18\frac{1}{4}$, which was $\frac{4}{5}$ of what he gave for it. What did he give for it?

148. Mr. Bracket sold a coat and gained $1.32, which was $\frac{1}{8}$ of what it cost him. What did it cost him?

149. If 5 yards of cloth cost $2.25, what will $7\frac{1}{2}$ yards cost?

150. If 14 cedar posts cost $8\frac{3}{4}$, what will 10 cost?

151. If $8\frac{1}{2}$ pounds of lead cost $1.70, what will $5\frac{3}{4}$ pounds cost?

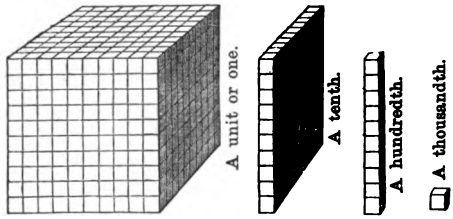
152. A man sold a firkin of butter for $16.80, and gained $\frac{1}{3}$ of what it cost him. What did it cost him?

153. On counting his money, Mr. Gould found he had $36.72. If this was $\frac{2}{3}$ as much as he had spent, how much had he spent? How much had he at first?

SECTION IX.

DECIMAL FRACTIONS.

259. *a.* If a block be divided into ten equal parts, what is each of the parts called?



b. If one of these tenths be divided into ten equal parts, what will each of these parts be called? What is $\frac{1}{10}$ of $\frac{1}{10}$?

c. If one of these hundredths be divided into ten equal parts, what will each of these parts be called? What is $\frac{1}{10}$ of $\frac{1}{100}$?

d. How many tenths make a unit?

e. How many hundredths make a tenth?

f. How many thousandths make a hundredth?

g. What part of a tenth is a hundredth?

h. What part of a hundredth is a thousandth?

One tenth of a thousandth is a ***ten-thousandth;*** one tenth of a ten-thousandth is a ***hundred-thousandth;*** one tenth of a hundred-thousandth is a ***millionth;*** and so on.

260. The fractions, *tenths, hundredths, thousandths,* etc., are ***decimal fractions,*** or ***decimals.***

To read and write Decimal Fractions.

261. Decimal fractions may be written like common fractions, with denominators. But they are usually written in the same way that integers are written. To distinguish

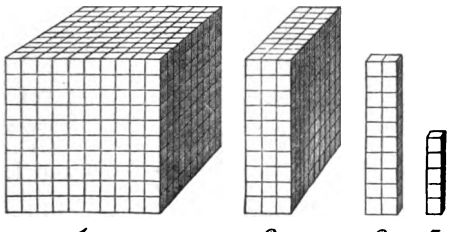
them from integers, a point (.), called a ***decimal point,*** is put at the left of the tenths' place.

Thus we write

One tenth	0.1	Three tenths.....	0.3
One hundredth	0.01	Two hundredths	0.02
One thousandth	0.001	Five thousandths.....	0.005

Three tenths and two hundredths, or thirty-two hundredths.... 0.32

Two hundredths and five thousandths, or twenty-five thousandths 0.025



Three tenths, two hundredths, and five thousandths, or three hundred twenty-five thousandths 0.325

One and three hundred twenty-five thousandths 1.325

It will be seen that in this method of writing decimals it is the numerator only that is written. How is the denominator indicated? Ans. *The denominator is indicated by the place of the right-hand figure,* counting from the decimal point.

262. The method of writing decimals is more fully shown by the following table, which is merely an extension of the table given on page 73.

Units.	Decimal Point.	Tenths.	Hundredths.	Thousandths.	Ten-thousandths.	Hundred-thousandths.	Millionths.
0	.	7	4	3	8	9	6

In which place from the right of the decimal point are ten-thousandths written? hundred-thousandths? millionths?

263. Exercises.

Read the following:

a.	*b.*	*c.*	*d.*	*e.*
0.4	0.04	5.	0.001	41.5
0.9	0.36	0.5	0.568	0.06
4.6	8.88	3.21	3.704	50.209
3.2	7.59	6.13	20.032	0.006
7.7	3.40	25.07	6.006	13.013

f.	*g.*	*h.*	*i.*
0.0001	5.2724	68.5	1324.
0.0045	0.3662	0.007	132.4
0.0182	0.3079	2.904	13.24
0.3247	17.3761	0.8764	1.324
26.0026	42.6305	279.0017	0.1324

Turn to page 129, and read the numbers written in H and I.

264. Examples for the Slate.

Write in figures the following numbers:

1. Eight tenths.
2. Seven hundredths.
3. Twelve thousandths.
4. Thirty-four hundredths.
5. Eight ten-thousandths.
6. 4 units and 5 hundredths.
7. 274 units and 8 tenths.
8. 906 hundred-thousandths.
9. 4 units and 7 ten-thousandths.
10. 2148 hundred-thousandths.

Let the teacher dictate other decimals for the pupils to write.

REDUCTION OF DECIMALS.

To change a Decimal to lower Denominations.

265. *a.* Change 3 to tenths. *b.* 0.5 to hundredths; to thousandths.

WRITTEN WORK.

$3 = 3.0$ read 30 tenths.
$0.5 = 0.50$ read 50 hundredths.
$0.5 = 0.500$ read 500 thousandths.

To express a decimal fraction in any lower denomination, *Annex zeros to the given expression until the place of the required denomination is reached.*

266. Examples for the Slate.

11. Change 7 to tenths; to hundredths; to thousandths.
12. Change 5 to thousandths; 0.25 to ten-thousandths;

To change Decimals to Common Fractions.

267. *c.* Change 0.5 to a common fraction in its simplest form.

WRITTEN WORK.

(*c.*) $0.5 = \frac{5}{10} = \frac{1}{2}$.

To change a decimal to a common fraction, *First write the fraction with its denominator; then change the common fraction to its smallest terms.*

268. Examples for the Slate.

Change the following to common fractions in their simplest forms:

(13.) 0.2; 0.4; 0.5; 0.6; 0.7; 0.05; 0.15; 0.25; 0.75.
(14.) 0.125; 0.375; 0.625; 0.875.

To change Common Fractions to Decimal Fractions.

269. *d.* Change $\frac{3}{8}$ to a decimal fraction.

WRITTEN WORK.

(*d.*) 8) 3.000
0.375

The fraction $\frac{3}{8}$ is the same as $\frac{1}{8}$ of 3, or $\frac{1}{8}$ of 3.000, which is found by dividing 3.000 by 8 in the usual way.

To change a common fraction to a decimal, *Express the numerator in tenths, hundredths, thousandths, etc., by annexing as many zeros as may be required, and then divide it by the denominator.*

270. Examples for the Slate.

Change

(15.) $\frac{1}{2}$ to tenths. (19.) $\frac{1}{4}$ to hundredths. (23.) $\frac{1}{8}$ to thousandths.
(16.) $\frac{1}{5}$ to tenths. (20.) $\frac{3}{4}$ to hundredths. (24.) $\frac{5}{8}$ to thousandths.
(17.) $\frac{3}{5}$ to tenths. (21.) $\frac{7}{10}$ to hundredths. (25.) $\frac{7}{8}$ to thousandths.
(18.) $\frac{4}{5}$ to tenths. (22.) $\frac{5}{6}$ to hundredths. (26.) $\frac{1}{8}$ to thousandths.

For other examples, see page 129.

ADDITION OF DECIMALS.

271. Examples for the Slate.

Write and add the following:

a.	WRITTEN WORK.
5 and 7 tenths,	5.7
13 and 8 tenths,	13.8
9 and 14 hundredths,	9.14
6 and 27 hundredths.	6.27
Ans.	34.91

In adding decimals, *Add as in integers, fixing the decimal point in the answer as soon as the tenths are added.*

(27.) Add

3 and 7 tenths,
75 and 8 tenths,
6 and 43 hundredths,
16 and 9 hundredths.

(28.) Add

68 and 72 hundredths,
3 and 9 tenths,
43 and 46 hundredths,
7 and 7 hundredths.
200 and 6 hundredths.

(29.) Add

8 and 19 hundredths,
56 and 9 tenths,
56 hundredths,
24 and 8 hundredths,
9 tenths.

(30.) Add

5 and 126 thousandths,
14 and 374 thousandths,
276 and 11 thousandths,
489 thousandths,
6 and 108 thousandths.

(31.) Add

1 and 926 thousandths,
754,
37 and 47 hundredths,
8 and 78 thousandths.

(32.) Add

5 and 4763 ten-thousandths,
21 and 2131 ten-thousandths,
3245 ten-thousandths,
9 and 875 ten-thousandths,
47 and 25 thousandths.

(33.) Add

14 and 3748 ten-thousandths,
4 and 93 hundredths,
7 and 872 thousandths,
5 and 8 tenths,
9875 ten-thousandths.

(34.) Add

7 and 1 hundredth,
5625 ten-thousandths,
875 thousandths,
46 and 97 ten-thousandths,
8 and 1008 ten-thousandths.

35. Add twelve and three tenths; seventy and thirty-five hundredths; two thousand three hundred forty-seven ten-thousandths; and seventy-eight thousandths.

36. Add forty-seven and three hundred seventy-eight thousandths; twenty-six ten-thousandths; four and nineteen hundredths; and sixty-six thousand six hundred and sixty-six hundred-thousandths.

37. Turn to page 106, and add the numbers written in Exercises *a* to *i*.

For other examples in Addition, see page 129.

SUBTRACTION OF DECIMALS.

272. Examples for the Slate.

a. From 2 and 75 hundredths take 928 thousandths.

WRITTEN WORK.

2.75
0.928
Ans. 1.822

In subtracting decimals, *Subtract as in integers, fixing the decimal point in the remainder as soon as the tenths are subtracted.*

38. From 14 and 5 tenths take 75 hundredths.

39. From 7 and 43 hundredths take 647 thousandths.

40. From 675 thousandths take 497 thousandths.

41. Subtract 445 thousandths from 2 and 4 tenths.

42. Subtract 66 thousandths from 6 and 6 tenths.

43. Subtract 9 ten-thousandths from 9 hundredths.

Find the difference of the following:

(44.)	(45.)	(46.)	(47.)	(48.)	(49.)
4.075	75.09	10.066	0.202	27.9384	1.11
2.439	7.446	0.0066	0.0202	3.42853	0.9999

273. Miscellaneous Examples.

50. A dozen lemons cost $0.38; four pounds of sugar, $0.36; and three pounds of crackers, $0.25. What did all cost?

51. One eighth of a dollar is $0.125; five eighths is $0.625. What is the sum expressed decimally? What is the difference?

52. If from a cask containing 15.4 gallons of ink 7.875 was drawn out, how much remained?

53. If the ages of John's grandfathers are 71.125 and 72.5 years, and of his grandmothers 69.33 and 70.25 years, what is the sum of all their ages?

54. What is the difference between the ages of John's grandfathers? between the ages of his grandmothers?

For other examples in Subtraction, see page 129.

MULTIPLICATION OF DECIMALS.

To multiply a Decimal by an Integer.

274. *a.* Multiply 0.3 by 4; 0.03 by 4; 0.003 by 4.

WRITTEN WORK.

0.3	0.03	0.003
4	4	4
1.2	0.12	0.012

Three tenths multiplied by 4 is 12 tenths; three hundredths multiplied by 4 is 12 hundredths; three thousandths multiplied by 4 is 12 thousandths.

In multiplying a decimal by an integer, how many places for decimals do you point off in the product?

275. Examples for the Slate.

55. Multiply 0.4 by 9; 0.04 by 9; 0.004 by 9.

56. Multiply 1.2 by 12; 0.12 by 12; 0.024 by 12.

57. Multiply 7.5 by 15; 6.25 by 5; 87.5 by 20.

58. What is the cost of 5 pounds of butter at \$0.3 per pound? at \$0.4? at \$0.35? at \$0.40?

59. There are 16.5 feet in one rod. How many feet in a mile, or 320 rods?

To multiply an Integer by a Decimal.

276. *b.* What is $\frac{1}{10}$ of 7? $\frac{1}{10}$ of 9? $\frac{1}{100}$ of 8? $\frac{1}{100}$ of 25?

c. Multiply 11 by 0.1; by 0.01; by 0.12.

WRITTEN WORK.

11	11	11
0.1	0.01	0.12
1.1	0.11	1.32

To multiply 11 by 0.1 is to take 1 tenth of it, which we express by placing the decimal point so that the figures 11 may express tenths, thus, 1.1.

To multiply 11 by 0.01 is to take 1 hundredth of it, which we express by placing the decimal point so that the figures may express hundredths; thus, 0.11.

To multiply 11 by 0.12 is to take 12 hundredths of it. One hundredth of 11 is 0.11, and 12 hundredths is 12 times 0.11, which equals 1.32.

Ans. 1.32.

In multiplying an integer by a decimal, how many places for decimals do you point off in the product?

277. Examples for the Slate.

60. Multiply 9 by 0.2; by 0.02; by 0.05.

61. Multiply 12 by 0.6; by 0.08; by 0.008.

62. Multiply 25 by 0.12; by 0.012; by 1.2.

63. At \$8 a ton for coal, what cost 0.9 of a ton? 0.09 of ton? 5.4 tons? 7.23 tons?

To multiply a Decimal by a Decimal.

278. *d.* What is $\frac{1}{10}$ of $\frac{1}{10}$? $\frac{1}{10}$ of $\frac{3}{10}$? $\frac{2}{10}$ of $\frac{3}{10}$? $\frac{5}{10}$ of $\frac{7}{10}$?

e. What is $\frac{1}{10}$ of $\frac{3}{100}$? 0.1 of 0.05? 0.4 of 0.06?

f. What is $\frac{1}{100}$ of $\frac{5}{10}$? 0.02 of 0.7? 0.06 of 0.9?

g. Multiply 0.5 by 0.9. *h.* Multiply 0.5 by 0.09.

WRITTEN WORK.	
(*g.*)	(*h.*)
0.5	0.5
0.9	0.09
0.45	0.045

(*g.*) To multiply 0.5 by 0.9 is to take 9 tenths of 5 tenths. One tenth of 5 tenths is 5 hundredths, and 9 tenths is 9 times 5 hundredths, which is 45 hundredths, 0.45.

(*h.*) In the same way we find that 0.5 multiplied by 9 hundredths gives 45 thousandths. As it requires three decimal places to write thousandths, and as in 45 there are but 2, we supply the deficiency by prefixing a zero before placing the decimal point, thus, 0.045.

In multiplying a decimal by a decimal, how many decimal places do you point off in the product?

279. In multiplying by decimals, *Multiply as in integers, and point off as many places for decimals in the product as there are decimal places in the multiplicand and multiplier counted together.*

280. Examples for the Slate.

64. Multiply 11.6 by 4.
65. Multiply 1.16 by 7.
66. Multiply 17.07 by 11.
67. Multiply 106 by 0.3.
68. Multiply 20 by 1.71.
69. Multiply 3.21 by 28.
70. Multiply 30.2 by 1.4.
71. $30.04 \times 0.105 = ?$
72. $920.8 \times 706.1 = ?$
73. $3.007 \times 0.005 = ?$
74. $5005. \times 0.001 = ?$
75. $88.04 \times 36 = ?$
76. $0.325 \times 0.018 = ?$
77. $0.0101 \times 4.16 = ?$

78. At \$0.62½ (\$0.625) each for hats, what is the cost of 10 hats? of 2 dozen?

79. At \$0.20 a square foot for land, what is the cost of 1000 square feet? of 2500?

80. What cost 12½ yards of cloth at \$1.875 a yard?

81. What must be paid for 100 feet of land at \$0.06¼ a foot?

82. There are 30.25 square yards in a square rod. How many square yards are there in 160 square rods or an acre?

DIVISION.

To divide a Decimal by an Integer.

281. *a.* What is 1 third of 0.6? 1 fourth of 0.8? of 1.2?

b. What is 1 half of 0.04? 1 fifth of 0.25? 1 eighth of 0.56?

c. Divide 13.75 by 5. *d.* Divide 10.272 by 4.

WRITTEN WORK.

(*c.*)	(*d.*)
5) 13.75	4) 10.272
2.75	2.568

In Example *c*, the dividend has two places for decimals, so in the quotient we mark off two places for decimals.

In Example *d*, the dividend has three places for decimals, so in the quotient we mark off three places for decimals.

In dividing a decimal by an integer, how many places do you point off for decimals in the quotient?

282. Examples for the Slate.

83. Divide 6.24 by 12.
84. Divide 288.9 by 9.
85. Divide 91.05 by 15.
86. Divide 1.576 by 8.
87. Divide 112.84 by 11.
88. Divide 14.607 by 27.

To divide Integers, carrying the Division to Decimals.

283. *e.* Find 1 fourth of 13.

WRITTEN WORK.

4) 13.00
3.25

In Example *e*, after the integer is divided there is a remainder of 1. This we change to 10 tenths. One fourth of 10 tenths is 2 tenths, and 2 tenths remain, which are equal to 20 hundredths. One fourth of 20 hundredths is 5 hundredths. The entire quotient is 3.25.

In the following examples carry the division to decimals:

89. Find 1 fifth of 157.
90. Find 1 sixth of 879.
91. Find 1 fourth of $927.
92. Find 1 eighth of $721.
93. If 8 hammocks cost $7, what is the cost of 1 hammock?
94. If 4 dozen hats cost $78, what is the cost of 1 hat?

284. *f.* Divide 348 by 71.

WRITTEN WORK.

71) 348 (4.901...
284
640
639
100
71
29

We divide as in Example *e* till we have thousandths in the quotient and there is still a remainder. We might keep on dividing, and thus come to greater accuracy in the quotient. But it is not generally necessary to carry the division beyond thousandths.

In the following examples carry the division to thousandths, and if there is a remainder then, indicate that there is one by dots, as in the answer to Example *f*.

95. How many are $4321 \div 9$?
96. How many are $3214 \div 12$?
97. How many are $9486 \div 19$?
98. Divide 876.1 by 8.
99. Divide 13.47 by 17.
100. Divide 9841 by 21.

To divide an Integer or a Decimal by a Decimal.

285. *g.* How many times are 0.3 contained in 0.6? 0.4 in 0.8? 0.6 in 1.2? 0.9 in 2.7?

h. How many times are 0.03 contained in 0.06? 0.08 in 0.16? 0.12 in 1.44?

i. Divide 0.006 by 0.002; 0.018 by 0.003; 0.024 by 0.012.

286. *j.* Divide 35 by 0.7. *k.* 1.44 by 0.8.

WRITTEN WORK.

(*j.*)	(*k.*)
0.7) 35.0$_{\wedge}$	0.8) 1.4$_{\wedge}$4
50	1.8

Before dividing by a fraction, the dividend must be expressed in the same denomination as the divisor. In Example *j*, the divisor is a number of tenths; the dividend expressed in tenths is 35.0 (350 tenths). 350 tenths divided by 7 tenths gives the same quotient as 350 divided by 7, which is 50. *Ans.* 50.

In Example *k*, the divisor is a number of tenths; the dividend expressed in tenths is 14.4 tenths (the denomination may be indicated, as in the written work, by a caret). 14.4 tenths divided by 8 tenths gives the same quotient as 14.4 divided by 8, which is 1.8. *Ans.* 1.8.

In Example *j*, how did you prepare the dividend to divide? How did you then divide?

In Example *k*, how did you prepare the dividend to divide? How did you indicate the denomination tenths in the dividend?

287. To divide by decimals: 1. *Express the dividend in the same denomination as the divisor by putting a mark as many places to the right of the decimal point as there are decimal places in the divisor.*

2. *Divide as if the divisor were an integer, fixing the decimal point of the quotient when the terms of the dividend have been used as far as the mark.*

288. Examples for the Slate.

101. Divide 315 by 0.7.
102. Divide 86.1 by 0.08.
103. Divide 70.32 by 0.38.
104. Divide 0.172 by 0.12.
105. Divide 0.5307 by 0.1.
106. Divide 1000 by 0.001.
107. Divide 90.09 by 0.071.
108. Divide 8.64 by 71.6.
109. Divide 538.1 by 4.001.
110. Divide 3.027 by 21.1.
111. How many tops at $0.02 apiece can be bought for $2?
112. How many rods, each 16.5 feet, are there in 100 feet?
113. At $0.125 a quart for cherries, how many quarts can he bought for $0.25? for $0.875? for $1.25? for $3?

For other examples in Multiplication and Division, see page 129.

SECTION X.

PERCENTAGE.

Oral Examples.

289. ***a.*** One is what part of 2? of 3? of 4? of 5? of 10?

b. One is what part of 20? of 30? of 50? of 100?

c. What part of 100 is 1? is 2? 3? 4? 5? 10? 20?

Suggestion. One is $\frac{1}{100}$ of a hundred; 2 is $\frac{2}{100}$; 3 is $\frac{3}{100}$; and so on.

290. Any number of hundredths of a thing or number is a ***per cent*** of that thing or number.

Thus, $\frac{2}{100}$ of a number is 2 per cent of it.

d. What per cent of a number is $\frac{5}{100}$ of it? $\frac{8}{100}$? $\frac{50}{100}$? $\frac{100}{100}$?

e. To take 20 per cent of a number is to take how many hundredths of the number?

f. How many hundredths of a number is 10 per cent of it? 25 per cent of it?

To express a given Per Cent.

291. The sign % is used for the words "per cent."

Thus, 3 % means 3 per cent.

292. Any per cent may be expressed as a common fraction, as a decimal, or with the sign for per cent, %. Thus,

1	per cent may be expressed	$\frac{1}{100}$,	0.01,	or 1%.
5	per cent " "	$\frac{5}{100}$,	0.05,	or 5%.
$8\frac{1}{3}$	per cent " "	$\frac{8\frac{1}{3}}{100}$,	$0.08\frac{1}{3}$,	or $8\frac{1}{3}$%.
100	per cent " "	$\frac{100}{100}$,	1.00,	or 100%.
125	per cent " "	$\frac{125}{100}$,	1.25,	or 125%.
$\frac{1}{2}$	per cent " "	$\frac{\frac{1}{2}}{100}$,	$0.00\frac{1}{2}$,	or $\frac{1}{2}$%.

293. Examples for the Slate.

Express the following in the three forms given above:

(1.) 2 per cent. (3.) $5\frac{1}{2}$ per cent. (5.) 145 per cent.

(2.) 8 per cent. (4.) 100 per cent. (6.) $\frac{1}{3}$ per cent.

To change a Common Fraction to a Per Cent.

294. *a.* What per cent of a number is $\frac{1}{2}$ of it? *b.* $\frac{1}{3}$ of it?

WRITTEN WORK.

(*a.*)	(*b.*)
2) 100%	3) 100%
50%	33⅓%

(*a.*) Since any number equals 100 per cent of itself, $\frac{1}{2}$ of the number must equal $\frac{1}{2}$ of 100%, or 50%. *Ans.* 50%.

(*b.*) $\frac{1}{3}$ of the number must equal $\frac{1}{3}$ of 100%, or 33⅓%. *Ans.* 33⅓%.

295. Oral Examples.

You will need to practise on these exercises till you can give the answers at sight.

c. What per cent of a number is $\frac{1}{4}$ of it? $\frac{1}{5}$? $\frac{1}{12}$? $\frac{1}{20}$? $\frac{1}{25}$? $\frac{1}{50}$? $\frac{1}{8}$? $\frac{1}{6}$?

d. What per cent of a number is $\frac{3}{5}$ of it? $\frac{3}{4}$? $\frac{3}{10}$? $\frac{3}{8}$? $\frac{4}{5}$? $\frac{5}{6}$? $\frac{3}{8}$? $\frac{5}{8}$? $\frac{7}{8}$? $\frac{3}{10}$? $\frac{7}{10}$? $\frac{9}{10}$?

e. What per cent of a number is $\frac{3}{20}$ of it? $\frac{7}{20}$? $\frac{9}{20}$? $\frac{9}{25}$? $\frac{7}{25}$? $\frac{3}{50}$? $\frac{7}{50}$?

296. From what you have learned in the preceding exercises, you may tell what part of a number the following per cents are, giving the parts in their smallest terms, thus, "10%, $\frac{1}{10}$; 20%, $\frac{1}{5}$," etc.

f. 10%; 20%; 30%; 50%; 25%; 75%; 80%; 90%; 12½%; 37½%; 62½%; 87½%; 33⅓%; 66⅔%; 16⅔%.

To find a Number that is a given Per Cent of another Number.

Oral Examples.

297. *g.* What number is 7% of $800?

Solution. — 7% of $800 is $\frac{7}{100}$ of $800, or $56. *Ans.* $56.

A number found by taking a number of hundredths of another number is a ***percentage*** of that number. Thus, in the example above, $56 is a percentage of $800.

h. What is 4% of 300 men? of $500? of 1000?

i. What is 10% of $50? of $25? of $80?

j. What is 1% of 2000? 3%? 5%? $6\frac{1}{4}$%?

k. What is 1% of \$1200? $\frac{1}{2}$%? $\frac{1}{3}$%? $\frac{1}{4}$%? 2%? $2\frac{1}{2}$%?

l. What is 1% of \$800? 100%? 101%? 102%? 110%?

m. A farmer raised 400 bushels of corn, and sold 50% of it. How many bushels did he sell?

n. In a school of boys and girls containing 300 pupils, 70% were girls. How many were boys?

298. Examples for the Slate.

a. What is 15% of \$946? *b.* $8\frac{1}{3}$% of \$87.60?

WRITTEN WORK.

(*a.*)	(*b.*)
946	87.60
0.15	$0.08\frac{1}{3}$
4730	2920
946	70080
141.90	7.3000

By what do you divide a number to find 1% of it?

Having found 1%, how do you find any number of times 1%?

Does it make any difference which operation you perform first?

(7.) What is 6% of \$469?
(8.) 8% of 975?
(9.) 9% of 1800?
(10.) 16% of 465?
(11.) 25% of 6800 men?
(12.) 42% of 876 miles?
(13.) $1\frac{1}{10}$% of 873 bushels.
(14.) $\frac{1}{8}$% of 120 acres?
(15.) $\frac{2}{3}$% of 784 pounds?
(16.) $\frac{3}{10}$% of 1600 days?
(17.) $\frac{4}{5}$% of 750 words?
(18.) 40% of 7.45?
(19.) 60% of 10.5 tons?
(20.) $18\frac{3}{4}$% of 960 barrels?

21. Of a flock of 175 sheep 4% were killed by dogs. How many were killed?

22. A stable-keeper bought two carriages; for the first he paid \$275, and for the second 75% as much as for the first. What did he pay for both?

23. A landlord has lowered his rents 20%. What will he now charge for tenements that have rented for \$200 a year? for \$425? for \$12.50 a month?

24. What does a man receive for a house-lot which cost \$1524 by selling it for $8\frac{1}{3}$% more than it cost?

25. What prices must be charged for four kinds of tea costing 36¢, 48¢, 54¢, and 63¢ per pound to gain $16\frac{2}{3}$%?

To find a Number when a Per Cent of it is given.

Oral Examples.

299. *a.* 20% of a number is $40. What is the number?

Solution. — Since 20% of the number sought is $40, 1% of the number sought is $\frac{1}{20}$ of $40, which is $2, and 100% of the number sought is 100 times $2, which is $200. *Ans.* $200.

b. 60 pounds is 6% of how many pounds?

c. $24 is 8% of how many dollars? $30 is 12% of how many dollars?

d. 90 is 10% of what number? 220 is 11% of what number?

e. 120 bushels is 60% of what number? $80 is 40% of what number?

f. John has 12¢, which is 6% of what Charles has. How much has Charles?

g. A man gained $20 by selling his watch for 10% more than it cost him. How much did it cost?

h. Mary spelt 98% of the words given out to her class and missed the rest, which was 6 words. What per cent did she miss? How many words were given out?

300. Examples for the Slate.

a. $250.50 is 25% of what number?

WRITTEN WORK.

$$\frac{250.50 \times 100}{25} = 1002.$$

Ans. $1002.

How do you find 1% of a number when a number of per cents are given? When you have found 1% of a number, how do you find the whole number?

(26.) 74 is 37% of what?

(27.) 150.50 is 5% of what?

(28.) $4.86 is 10% of what?

(29.) $649.33 is 11% of what?

(30.) $75 is 37½% of what?

(31.) $94.50 is 6¼% of what?

(32.) $31.25 is 125% of what?

(33.) $87.50 is 62½% of what?

34. A man expends 70% of his income and saves $369. What is his income?

35. I drew from a bank $1485, which was 45% of what was left. What was the sum in at first?

To find what Per Cent one Number is of another.

Oral Examples.

301. ***a.*** What per cent of 50 is 7 ?

Solution. — 7 is $\frac{7}{50}$ of 50. $\frac{7}{50}$ equals $\frac{14}{100}$, or 14%. *Ans.* 14%.

b. What per cent of 20 is 1 ? is 3 ? is 9 ?

c. What per cent of 10 is 1 ? is 3 ? is 7 ?

d. What per cent of 25 is 2 ? is 3 ? is 6 ?

e. What per cent of 80 is 20 ?

Solution. — 20 is $\frac{1}{4}$ of 80. $\frac{1}{4}$ equals $\frac{25}{100}$, or 25%. *Ans.* 25%.

f. What per cent of 40 is 8 ? of 16 is 4 ? of 12 is 6 ? of 25 is 5 ?

g. A boy had 20 plums, and gave away 7 of them. What per cent did he give away ? What per cent did he keep ?

h. A man hired $50, and paid $3 for the use of it. What per cent did he pay ?

i. A boy who weighed 80 pounds lost 10 pounds by sickness. What per cent did he lose ?

j. A watch which cost $20 was sold for $15. How many dollars were lost ? What per cent was lost ?

302. Examples for the Slate.

k. What per cent of $50 is $3.25 ?

WRITTEN WORK.

50) 3.25 (0.06$\frac{25}{50}$ = 6$\frac{1}{2}$%.
3 00
25

To find what per cent one number is of another, *Divide the percentage by the number it is a percentage of, carrying the division to hundredths.*

36. What per cent of $480 is $40 ?

37. Of 16 days is 14 days ?

38. Of 35 cents is 8 cents ?

39. Of 15 tons is 2 tons ?

40. Of 1 score is 1 dozen ?

41. If I sell a horse for $100 which cost $150, what per cent do I lose ?

42. Flour which was bought at $4.80 per barrel is sold at $6.00. What is the per cent of gain ?

PROFIT AND LOSS.

Oral Examples.

303. *a.* How much money is gained by selling a cow for 10% above cost, the cost being $60? $80?

b. How much money is lost on a horse which cost $200 by selling him at a loss of 10%? of 20%? of 50%?

c. At what price must books which cost $4 apiece be sold to gain 50%? 25%? 20%? 10%?

d. A man bought butter at 20 cents a pound. At what price must he sell it to lose 25%? 20%? 30%?

e. A man sold a cow for $66, and gained 10%. What did she cost him?

Solution. — Since he gained 10%, $66 must be 110% of the cost. If $66 is $\frac{110}{100}$ or $\frac{11}{10}$ of the cost, $\frac{1}{10}$ of the cost must be $\frac{1}{11}$ of $66, or $6, and the whole cost must be 10 times $6, or $60. *Ans.* $60.

f. What must have been paid a bushel for wheat, if by selling it for $1.50 there was a gain of 25%? of 50%?

g. A man sold a wagon for $48, and lost 20%. What was the cost of the wagon?

Suggestion. Since he lost 20%, $48 is 80%, or $\frac{4}{5}$ of the cost.

h. What was the cost of knives which sold for 27 cents each at a loss of 10%? 25%?

i. What per cent would be gained by selling a plough for $10 which cost $8?

Suggestion. $2 was gained on $8, which was $\frac{2}{8}$ or $\frac{1}{4}$ of 8. $\frac{1}{4}=25\%$.

j. What per cent is gained if spades costing 80 cents are sold for $1.00? for $1.20? for $1.30?

304. The difference between the cost of goods and the price at which they are sold is a ***profit*** or ***loss***.

305. Examples for the Slate.

43. Cloth which was bought at $7 a yard was sold at a gain of 12%. What was received for it?

44. For what must I sell 150 cords of wood which cost $3.50 a cord to gain 8%?

45. By selling corn at 25 % profit a merchant gained $462.50. What did the corn cost him?

46. From a barrel of kerosene 15½ gallons leaked out. If this was 30% of the contents of the barrel, how many gallons did the barrel contain at first?

47. The expenses of a family in 1877 were $588. In 1878 they were 7½% less. What were their expenses in 1878?

48. A shoe-dealer sold boots at $4.50 a pair at a gain of 20% on what they cost him. What did they cost?

49. If 120% of a number is $4.50, what is the number?

50. A man sold coal at $5.50, and lost 23% of the cost. What did his coal cost?

51. What per cent is gained by selling nutmegs at 56 cents a pound which cost 48 cents a pound?

52. A, having failed, pays B $280 instead of $480, which he owed him. What per cent does B lose?

306. COMMISSION, INSURANCE, TAXES, ETC.

NOTE. The terms Commission, Insurance, Taxes, etc., should be explained by the teacher. A full treatment of these subjects will be found in the Franklin Written Arithmetic.

53. An agent sold a house for $2500. If he received a commission of 4% for selling, how much did he receive?

54. A lawyer received 5% commission for collecting a bill of $375. What was his commission?

55. An architect received a commission of $140.30 for planning and superintending the building of a house. If his commission was 3%, what was the cost of the house?

56. A man had his house insured for $2000. If he paid 1½% of this sum for the insurance, what did he pay?

57. A man is taxed for property amounting to $4760. If his tax is 1¼% of this amount, what is his tax?

58. A broker bought for a man 5 shares of railroad stock worth $100 a share, and charged ⅛% commission. What was his charge?

SECTION XI.

INTEREST.

307. A man lent me $200 for 1 year. At the end of the year I paid him back $200, with 5% of $200 for the use of the money. How much did I pay for the use of the money? How much in all? *Ans.* $10 for the use; $210 in all.

Money paid for the use of money is ***interest.*** The money for the use of which interest is paid is the ***principal.***

The sum of the principal and interest is the ***amount.***

Thus, in the above example $10 is the interest, $200 is the principal, and $210 is the amount.

308. To find the interest, a certain per cent of the principal is taken for each year. This per cent is called the ***rate per cent,*** or simply the ***rate.***

NOTE. In reckoning interest, it is customary to consider a year to be 12 months, and a month 30 days.

309. ***a.*** What is the interest of $200 for 2 years at 6%?

WRITTEN WORK.

200
0.06
12.00
2
24.00

6% is $\frac{6}{100}$, or 0.06. 0.06 of $200 is $12, the interest for 1 year.

The interest for 2 years is 2 times $12, or $24.

Ans. $24.

310. Oral Examples.

What is the interest

b. Of $100 at 6% for 2 years? 3 years?

c. Of $200 for 2 years at 3%? 4%? 5%? 7%? 12%?

d. Of $40 at 10% for 1 year? for 3 years 6 months?

SOLUTION.

$40 × 10% = $4. Int. for 1 y. (year).

$4 × 3½ = $14. " " 3 y. 6 mo. (months).

3 years 6 months equals 3½ years.

What is the interest of $60 at 10%

e. For 1 y. 6 mo.? *g.* For 1 y. 9 mo.?

f. For 1 y. 3 mo.? *h.* For 2 y. 8 mo.?

What is the interest of $120 at 10%

i. For 4 y. 2 mo.? *k.* For 2 y. 5 mo.?

j. For 4 y. 1 mo.? *l.* For 6 y. 7 mo.?

The Six Per Cent Method.

A general method of computing interest will be found on p. 142 of the Appendix.

311. Oral Examples.

a. At 6%, how many hundredths of the principal is the interest for 1 y.? for 2 y.? for 3 y.? *Ans.* 0.06; 0.12; 0.18.

At 6%, how many hundredths of the principal is the interest

b. For 6 y.? for 8 y.? 10 y.? 11 y.?

c. For 6 mo.? for 2 mo.?

Suggestion. Since the interest is 6% for 1 year, for 6 months, which is 1 half of a year, it is 3%, or 0.03; for 2 months, which is 1 sixth of a year, it is 1%, or 0.01; and so on.

At 6%, how many hundredths of the principal is the interest

d. For 2 mo.? 4 mo.? 8 mo.? 10 mo.?

e. For 1 y. 6 mo.? 2 y. 6 mo.? 3 y. 2 mo.?

f. For 4 y. 2 mo.? 3 y. 4 mo.? 5 y. 8 mo.?

g. For 1 mo.? 3 mo.? 5 mo.?

Suggestion. Since the interest is 1% for 2 months, for 1 month it is $\frac{1}{2}$%, or 0.005; for 3 months it is $1\frac{1}{2}$%, or 0.015; and so on.

At 6%, what part of the principal is the interest

h. For 3 mo.? 5 mo.? 7 mo.? 9 mo.? 11 mo.?

i. For 1 y. 1 mo.? 2 y. 1 mo.? 2 y. 3 mo.? 2 y. 2 mo.?

j. For 3 y. 1 mo.? 4 y. 5 mo.? 3 y. 6 mo.? 3 y. 7 mo.?

k. For 5 y. 10 mo.? 6 y. 9 mo.? 6 y. 8 mo.? 10 y. 11 mo.?

l. For 6 days? for 1 day? for 5 days? for 7 days?

Suggestion. Since the interest is 0.005 of the principal for 1 month or 30 days, for 6 days, which is 1 fifth of a month, it is 0.001; for 1 day it is 0.000$\frac{1}{6}$; for 5 days, 0.000$\frac{5}{6}$; for 7 days, 0.001$\frac{1}{6}$; and so on.

At 6%, what part of the principal is the interest

m. For 6 d. (days) ? 12 d. ? 18 d. ? 24 d. ? 1 mo. (30 d.) ?

n. For 1 d. ? 3 d. ? 9 d. ? 10 d. ? 11 d. ? 13 d. ?

o. For 15 d. ? 19 d. ? 25 d. ? 17 d. ? 21 d. ? 27 d. ?

312. Examples for the Slate.

a. At 6%, what part of the principal is the interest for 2 y. 5 mo. 14 d. ?

WRITTEN WORK.

0.12 for 2 years.
0.025 " 5 months.
$0.002\frac{1}{3}$ " 14 days.
$0.147\frac{1}{3}$ " 2 y. 5 mo. 14 d.

To find what part of the principal the interest is, *Take 6 times as many hundredths as there are years, 1 half as many hundredths as there are months, and 1 sixth as many thousandths as there are days.*

At 6%, what part of the principal is the interest

1. For 1 y. 1 mo. 1 d. ?
2. For 1 y. 3 mo. 5 d. ?
3. For 2 y. 6 mo. 9 d. ?
4. For 3 y. 8 mo. 23 d. ?
5. For 5 y. 7 mo. 7 d. ?
6. For 4 y. 5 mo. 25 d. ?
7. For 8 y. 4 mo. ?
8. For 6 mo. 10 d. ?
9. For 2 y. 10 d. ?
10. For 3 y. 5 mo. 14 d. ?

313. *b.* Find the interest of $120 at 6% for 3 y. 5 mo. 14 d.

WRITTEN WORK.

120
$0.207\frac{1}{3}$
40
840
240
24.880

At 6%, the interest for 3 years 5 months 14 days is $0.207\frac{1}{3}$ of the principal. $0.207\frac{1}{3}$ of $120 is $24.88.

Ans. $24.88.

To compute the interest on any sum at 6 %: (1) *Find the decimal that expresses the part which the interest is of the principal;* (2) *by this decimal multiply the principal.*

NOTE. In doing the following examples, get the answers to the nearest cent.

At 6%, what is the interest

11. Of $800 for 1 y. 4 mo. ?
12. Of $124 for 2 y. 7 mo. ?
13. Of $350 for 3 y. 6 mo. 9 d. ?
14. Of $488 for 5 y. 7 mo. 14 d. ?

314. 15. At 6%, what is the interest of $500 from Jan. 1, 1877, to March 7, 1878?

Suggestion. From Jan. 1, 1877, to Jan. 1, 1878, is 1 year; from Jan. 1 to March 1 is 2 months; and from March 1 to March 7 is 6 days. The whole time is 1 y. 2 mo. 6 d.

At 6%, what is the interest

16. Of $268.90 from July 15, 1877, to Sept. 25, 1878?

17. Of $1185 from May 3, 1878, to August 18, 1879?

18. Of $2000 from Oct. 25, 1878, to Jan. 29, 1879?

Suggestion. The whole time is 3 months 4 days.

At 6%, what is the interest

19. Of $87.50 from Sept. 14, 1877, to Feb. 20, 1878?

20. Of $380.70 from Sept. 17, 1877, to March 29, 1879?

21. Of $200 from Dec. 16, 1877, to May 10, 1878?

Suggestion. From Dec. 16, 1877, to *April 16*, 1878, is 4 months. From April 16 to April 30 is 14 days, and to May 10 is 10 days more, or 24 days. The whole time is 4 months 24 days.

At 6%, what is the interest

22. Of $95 from Nov. 26, 1878, to May 15, 1879?

23. Of $284.40 from Nov. 27, 1877, to Oct. 15, 1879?

24. At 6%, what is the amount of $7500 from Feb. 15, 1878, to Dec. 29, 1879?

Suggestion. To find the amount, add the principal to the interest.

At 6%, what is the amount

25. Of $496 from June 17, 1877, to Dec. 1, 1878?

26. Of $630.20 from Feb. 18, 1877, to July 5, 1878?

315. To find the Interest at any Per Cent other than 6%.

What is the interest of $285, for 3 y. 4 mo.

27. At 7%? 28. At 8%? 29. At 5%?

Suggestion. First find the interest at 6%; then to find the interest at 7%, add $\frac{1}{6}$ of this interest to itself, to find the interest at 8%, add $\frac{2}{6}$ or $\frac{1}{3}$. To find the interest at 5%, subtract $\frac{1}{6}$, and so on.

What is the interest of $156.60 for 3 y. 6 mo. 18 d.

30. At 4%? 31. At 8%? 32. At 10%?

What is the interest

33. Of $474 for 2 y. 11 mo. at 7%?

34. Of $2500 from Oct. 15, 1878, to July 27, 1879, at 10%?

What is the interest and amount

35. Of $318 for 11 mo. 90 d. at 5%?

36. Of $268 for 3 y. 8 mo. 27 d. at 8%?

37. Of $98.25 for 1 y. 5 mo. 1 d. at 7%?

What is the amount

38. Of $805.50 from Sept. 10, 1878, to Nov. 4, 1878, at 6½%?

39. Of $962.25 from Apr. 8, 1875, to Oct. 1, 1875, at 7%?

40. Of $500 from Jan. 1, 1878, to Jan. 15, 1879, at 7½%?

41. A note for $400 was on interest at 6%, from June 5, 1875, to June 5, 1876, when a payment of $124 was made to pay the interest due and part pay the note. How much then remained due?

42. If $300 was due on a note June 5, 1876, and was kept on interest till December 5, 1876, what was then required to pay the note and the interest due?

BANK DISCOUNT.

316. On the 7th of June, 1878, Mr. M. J. Oliver bought a horse of Mr. John Day, and agreed to pay $225 for it in 60 days, giving him the following written promise, called a

PROMISSORY NOTE.

$225. *Lincoln, June 7, 1878.*

Sixty days from date I promise to pay John Day, or order, Two Hundred Twenty-five Dollars; value received.

M. J. Oliver.

If Mr. Day wants the money before the 60 days are over, he can take the note to a bank and get the money by pay-

ing the bank the interest on it for the time to elapse before it is due, and for 3 days more.

The note is then said to be ***discounted.***

317. The interest paid to the bank is ***bank discount.***

318. The money received from the bank is called the ***proceeds.***

319. The three days for which interest is taken beyond the time named in the note are called ***days of grace.***

320. a. Find the discount on the above note July 7, 1878, the discount being 6%. Find the proceeds.

WRITTEN WORK.

$225
0.0055
1125
1125
$1.2375

Bank discount is interest for the specified time and of 3 days' grace.

The time from July 7 to the end of 60 days and grace is 33 days.

The interest of $225 for 33 days at 6% is $1.24, which is the discount. $225 less $1.24 equals $223.76, the proceeds of the note.

$225 − $1.24 = $223.76. *Ans.* $1.24, discount; $223.76, proceeds.

321. Examples for the Slate.

43. What is the bank discount of a note for $500, payable in 30 days, discount 6%. What are the proceeds?

NOTE. Add 3 days of grace to the specified time in all instances.

Find the bank discount and proceeds of a note

44. For $300, payable in 60 days, discount 6%.

45. For $75.80, payable in 90 days, discount 7%.

46. For $1300, payable in 4 months, discount 5%.

47. For $486.50, payable in 45 days, discount 8%.

48. For $275.48, payable in 3 months, discount 10%.

49. For $80, due July 15, and discounted June 15, at 6%.

50. For $700, due Aug. 29, and discounted Aug. 2, at 5%.

51. For $650, dated Aug. 25, payable in 2 months, and discounted Sept. 25, at 6%.

52. For $2000, dated Oct. 1, payable in 1 month, and discounted Oct. 16, at 9%.

322. DRILL TABLE No. 4.

Common Fractions.—Decimals.—Percentage.

Examples.	A	B	C	D	E	F	G
1.	$\frac{1}{2}$	$\frac{1}{4}$	$\frac{10}{18}$	$3\frac{5}{7}$	5	$19\frac{2}{5}$	26
2.	$\frac{1}{3}$	$\frac{2}{8}$	$\frac{27}{36}$	$5\frac{5}{8}$	16	$12\frac{5}{8}$	18
3.	$\frac{7}{8}$	$\frac{9}{16}$	$\frac{32}{48}$	$12\frac{5}{9}$	13	$16\frac{5}{12}$	32
4.	$\frac{13}{15}$	$\frac{5}{30}$	$\frac{7}{35}$	$8\frac{1}{7}$	19	$9\frac{2}{3}$	27
5.	$\frac{5}{6}$	$\frac{7}{12}$	$\frac{16}{48}$	$9\frac{2}{5}$	7	$11\frac{5}{9}$	93
6.	$\frac{7}{8}$	$\frac{3}{16}$	$\frac{15}{60}$	$12\frac{2}{3}$	12	$15\frac{7}{8}$	45
7.	$\frac{3}{4}$	$\frac{5}{12}$	$\frac{12}{36}$	$6\frac{2}{7}$	9	$8\frac{1}{9}$	38
8.	$\frac{11}{12}$	$\frac{3}{24}$	$\frac{21}{42}$	$14\frac{2}{5}$	18	$17\frac{1}{3}$	62
9.	$\frac{11}{15}$	$\frac{1}{30}$	$\frac{25}{75}$	$7\frac{1}{4}$	8	$12\frac{1}{5}$	21
10.	$\frac{5}{7}$	$\frac{4}{21}$	$\frac{14}{63}$	$11\frac{2}{15}$	21	$17\frac{1}{2}$	33
11.	$\frac{2}{3}$	$\frac{5}{9}$	$\frac{15}{75}$	$5\frac{3}{20}$	14	$8\frac{1}{6}$	54
12.	$\frac{8}{9}$	$\frac{4}{27}$	$\frac{16}{30}$	$9\frac{3}{5}$	8	$15\frac{1}{3}$	28
13.	$\frac{7}{10}$	$\frac{4}{50}$	$\frac{6}{45}$	$6\frac{5}{6}$	17	$12\frac{3}{5}$	72
14.	$\frac{4}{5}$	$\frac{4}{25}$	$\frac{14}{35}$	$8\frac{5}{9}$	9	$14\frac{1}{8}$	91
15.	$\frac{9}{11}$	$\frac{7}{44}$	$\frac{30}{48}$	$4\frac{3}{4}$	15	$6\frac{2}{5}$	19
16.	$\frac{7}{9}$	$\frac{8}{63}$	$\frac{32}{42}$	$7\frac{1}{8}$	24	$11\frac{2}{5}$	52
17.	$\frac{5}{8}$	$\frac{5}{24}$	$\frac{16}{36}$	$5\frac{7}{12}$	11	$13\frac{3}{20}$	83
18.	$\frac{10}{11}$	$\frac{1}{22}$	$\frac{45}{55}$	$12\frac{4}{9}$	13	$15\frac{4}{7}$	60
19.	$\frac{7}{12}$	$\frac{5}{36}$	$\frac{45}{60}$	$13\frac{1}{3}$	16	$20\frac{1}{5}$	48
20.	$\frac{6}{7}$	$\frac{3}{42}$	$\frac{21}{49}$	$11\frac{1}{21}$	7	$11\frac{1}{14}$	95
21.	$\frac{11}{12}$	$\frac{1}{24}$	$\frac{45}{108}$	$16\frac{3}{4}$	22	$19\frac{3}{7}$	35
22.	$\frac{3}{5}$	$\frac{6}{35}$	$\frac{20}{75}$	$7\frac{1}{3}$	10	$9\frac{3}{8}$	67
23.	$\frac{9}{10}$	$\frac{7}{60}$	$\frac{45}{105}$	$10\frac{1}{4}$	12	$11\frac{5}{11}$	75
24.	$\frac{4}{7}$	$\frac{16}{35}$	$\frac{26}{65}$	$8\frac{1}{2}$	25	$9\frac{1}{5}$	88
25.	$\frac{3}{4}$	$\frac{4}{12}$	$\frac{12}{42}$	$7\frac{4}{7}$	13	$13\frac{1}{5}$	42

DRILL TABLE No. 4

(*continued*).

Examples	H	I
1.	2.905	3.178
2.	3.061	50.817
3.	0.052	0.703
4.	38.04	209.63
5.	200.1	54.68
6.	0.05	3.0721
7.	708.1	0.1001
8.	362 3	0.0732
9.	52.12	9.246
10.	1.805	43 287
11.	100.2	0.016
12.	0.086	9.03
13.	702.4	3.0072
14.	0.7	0.245
15.	5 099	3.18
16.	80.06	5.4061
17.	0.004	10.18
18.	581.9	3.287
19.	600.7	540.91
20.	7.101	10.6
21.	71.01	4.8001
22.	19.06	13.704
23.	0.83	97.3001
24.	28.5	2.007
25.	16.035	191.06

Exercises upon the Table.

323. Common Fractions.

124. Change C to smallest terms.
125. Change D to improper fractions.
126. Add A and B.
127. Add A, C, and D.
128. Add D and F.
129. A − B.
130. G − F.
131. D − C.
132. F − D.
133. A × E.
134. G × B.
135. A × C.
136. A × D.
137. D × F.
138. A ÷ E.
139. E ÷ B.
140. A ÷ B.
141. D ÷ A.
142. F ÷ D.

324. Decimals.

143. Change A to a decimal (3 places).
144. Change B to a decimal (4 places).
145. Add H, I, and 2.9784.
146. From 1000 take H.
147. Find the difference of H and I.
148. Multiply I by 7.
149. Multiply 20 by H.
150. Multiply H by I.
151. Divide H by 4.
152. Divide 57 by H (3 places).
153. Divide I by H (4 places).

325. Percentage.

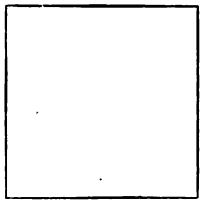
154. What is 5% of E dollars?
155. What is 7% of G dollars?
156. What is 8% of H dollars?
157. Find the interest of G dollars for 3 y. 6 mo. at 6%.
158. Find the interest of E hundred dollars for 4 y. 9 mo. 15 d. at 6%.
159. Find the interest of G hundred dollars for 2 y. 3 mo. 14 d. at 5%.

SECTION XII.

MENSURATION.

NOTE. For those who may study no higher Arithmetic, brief directions are here given for finding the areas of some surfaces and the volume of some solids. Further explanations and illustrations of these surfaces and solids should be given by the teacher, who is referred to the Franklin Written Arithmetic, pages 137 to 139; also pages 277 to 288.

SURFACES.



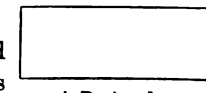
A Square.

326. A square each of whose sides is 1 inch long is a ***square inch;*** a square each of whose sides is 1 foot long is a ***square foot;*** etc.

327. The ***area*** of a surface is its contents reckoned in square units.

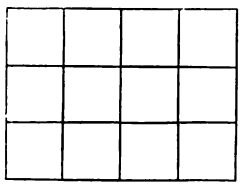
To find the Area of a Rectangle.

328. A ***rectangle*** is a four-sided figure which has all its corners square.



A Rectangle.

a. If a rectangle is 4 inches long and 1 inch wide, how many square inches does it contain? How many will it contain if it is 4 inches long and 2 inches wide? 3 inches wide?



b. What numbers will you multiply together to find the number of square units in any rectangle?

329. The area of a rectangle is found by *multiplying the number of units in the length by the number of like units in the breadth.* This is expressed for brevity as *multiplying the length by the breadth.*

330. Oral Exercises.

c. How many square inches are there in the surface of a slate that is 9 inches long and 8 inches wide? that is 12 inches (1 foot) long and 12 inches (1 foot) wide? Then how many square inches are there in a square foot?

d. How many square feet are there in a rectangle that is 5 feet long and 3 feet wide? How many square feet are there in a square whose sides are each 3 feet (1 yard) long?

331. Repeat the following table:

Square Measure.

144 square inches	= 1	square foot.
9 square feet	= 1	square yard.
$30\frac{1}{4}$ square yards *or* $272\frac{1}{4}$ square feet	= 1	square rod.
160 square rods	= 1	acre.
640 acres	= 1	square mile.

332. Examples for the Slate.

1. In a walk 20 feet long and 6 feet wide, how many square feet? how many square yards?

2. How many square yards of carpeting will it take to cover a floor that is 15 feet long and 14 feet wide?

3. In a piece of ground 200 feet long and 50 feet wide, how many square feet? how many square rods?

4. In a lot 250 feet long and 120 feet wide, how many square rods?

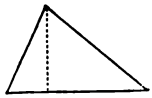
5. How many square feet of land in a lot 4 rods wide and 90 feet deep?

6. At 20¢ a square foot for land, what must be paid for a lot 100 feet deep and $3\frac{1}{2}$ rods on the front?

7. In a square field 60 rods on each side, how many acres?

8. What is the difference in the size of two lots, one containing 65 square rods, the other being 65 rods square, that is, 65 rods long and 65 rods wide?

TRIANGLES, POLYGONS, AND CIRCLES.

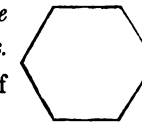


A Triangle.

333. A triangle is half a rectangle of the same base and height. Hence,

To find the area of a triangle, *Multiply the base by the height and divide the product by 2.*

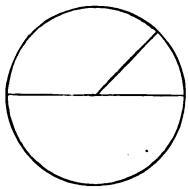
334. To find the area of a polygon, *Divide it into triangles and find the sum of their areas.*



A Polygon.

335. When the diameter of a circle is given,

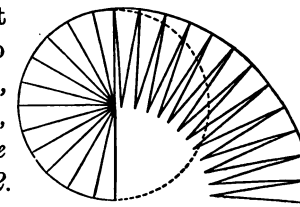
To find the circumference, *Multiply the diameter by 3.1416.*



A Circle.

336. A circle may be said to consist of triangles whose bases form the circumference, the height of the triangles being equal to the radius of the circle. Hence,

To find the area of a circle, *Multiply the circumference by the radius and divide the product by 2.*



337. Examples for the Slate.

8. I have a piece of woodland shaped like a triangle, the base of which is 100 rods long and the height 62 rods. How many acres does it contain?

9. How many square yards are there in a roof that is made up of 4 triangles, the base of each being 20 feet and the height 15 feet?

10. I have a tin-pail that measures 8 inches across the top. What is the distance round it? How many square inches must there be in a cover that will fit it?

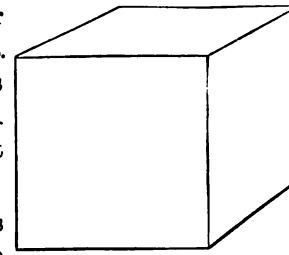
11. How many square feet of land in a circular flower-bed that measures 10 feet across?

SOLIDS.

338. A ***rectangular solid*** is a solid bounded by six rectangles. A brick is a rectangular solid body.

339. A ***cube*** is a rectangular solid bounded by six equal squares.

A cube each of whose edges is 1 inch long is a cubic inch. A cube each of whose edges is 1 foot long is a cubic foot, etc.

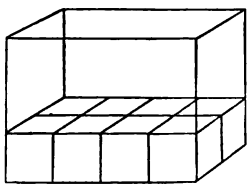


A Cube.

340. The ***volume*** of a solid is its contents reckoned in cubic units.

To find the Volume of a Rectangular Solid.

341. *a.* If a rectangular solid is 4 inches long and 2 inches wide, how many square inches must its base contain? (Art. 329.)



b. If the base of the solid contains 4 times 2, or 8 square inches, and it is 1 inch thick, how many solid or cubic inches must it contain? If it is 2 inches thick, how many cubic inches must it contain? If it is 3 inches thick, how many cubic inches must it contain?

What numbers have you multiplied together to find the volume of the rectangular solid 4 inches long, 2 inches wide, and 3 inches high?

342. The volume of any rectangular solid is found by *multiplying the number of units in the length by the number of like units in the breadth, and this product by the number of like units in the thickness.* This is expressed, for brevity, as *multiplying together the length, breadth, and thickness.*

343. Oral Exercises.

c. How many cubic inches in a rectangular solid 6 inches long, 5 inches wide and 3 inches thick? 4 inches long, 2 inches wide and 7 inches thick? 12 inches long, 12 inches wide and 1 inch thick?

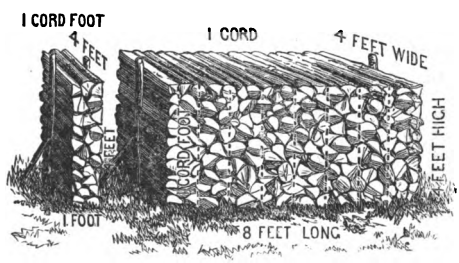
d. How many cubic feet in a cube 3 feet long, 3 feet wide and 3 feet thick, or in 1 cubic yard?

344. Repeat the following table:

Cubic Measure.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
128 cubic feet = 1 cord (*used in measuring wood*).

Wood is generally cut for the market into sticks 4 feet long, and laid in piles, so that the length of the sticks becomes the width of the pile. A pile 4 feet wide, 4 feet high, and 8 feet long, contains 1 cord.



One eighth of a cord is called 1 ***cord foot.*** 1 cord foot contains 16 cubic feet. (See illustration above.)

345. Examples for the Slate.

12. How many cubic inches will a box contain that is 11 inches long, 7 inches wide and 6 inches high, inside measure?

13. How many cubic inches in a beam 5 feet long, 3 inches thick and 4 inches wide?

14. A cubic foot of water weighs 62½ pounds. What is the weight of water that a cistern may contain which measures on the inside 4 feet in length, 3 feet in width and 4 feet in depth?

15. How many cords of wood in a pile 20 feet long, 4 feet wide and 4 feet high?

16. How many cords in a pile 30 feet long, 4 feet wide and 3½ feet high?

Lumber and Boards.

346. Sawed timber and boards, when 1 inch or less in thickness, are generally reckoned by the square foot of surface measure. When more than 1 inch in thickness, they are reckoned in proportion to their thickness. Thus,

2000 square feet, 1 inch or less thick = 2000 feet, board measure.
2000 square feet, 1½ inches thick = 3000 feet, board measure.
2000 square feet, 2 inches thick = 4000 feet, board measure.

And so on.

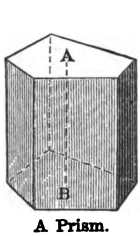
17. How many feet of boards, 1 inch thick, in a tight board fence, 2 rods long and 4 feet high?

18. How many feet board measure, in a plank 7 feet long, 2½ inches thick, 14 inches wide at one end and 10 inches wide at the other?

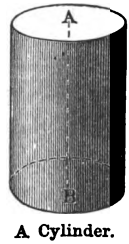
NOTE. First find the *average width* which equals one half the sum of the widths at the ends.

19. How many feet, board measure, in a pile of 12 boards, each board being ¾ of an inch thick, 14 feet long, 14 inches wide at one end and 11 inches wide at the other?

PRISMS, CYLINDERS, PYRAMIDS, CONES, AND SPHERES.



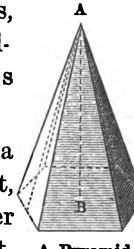
A Prism.



A Cylinder.

347. To find the volume of a prism or a cylinder, *Multiply the area of the base by the height.* Thus, if the area of the base of a prism is 15 square inches and the height 9 inches, the volume is 15 × 9, or 135 cubic inches.

348. A pyramid is 1 third of a prism of the same base and height, and a cone is 1 third of a cylinder of the same base and height.

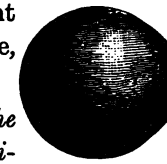


A Pyramid.

A Cone.

Hence, to find the volume of a pyramid or a cone, *Multiply the area of the base by the height and divide the product by 3.*

349. The surface of a sphere is equivalent to four great circles of the sphere. Hence, when the diameter is given,



A Sphere.

To find the surface of a sphere, *Find the area of a great circle of the sphere and multiply it by 4.*

350. A sphere may be regarded as made up of pyramids whose bases taken together form the surface of the sphere and whose height is the radius. Hence,

To find the volume of a sphere, *Multiply the surface by the radius and divide the product by 3.*

351. Examples for the Slate.

20. The base of a prism contains 4 square feet, and its height is 3 feet 6 inches. How many cubic feet does it contain?

21. How many cubic feet of water will a circular tub contain, that measures across the top and bottom $2\frac{1}{2}$ feet, and whose depth is $1\frac{1}{2}$ feet?

NOTE. First find the area of the bottom.

22. I have a paper weight made of flint glass in the shape of a pyramid, measuring 4 inches square on the base and 5 inches in height. A cubic inch of the glass weighs $1\frac{5}{12}$ ounces, what is the weight of the whole pyramid?

23. How many cubic feet in a conical hay-stack that measures across the base 16 feet, the height being 18 feet?

24. How many square inches are there in the surface of a foot-ball that is 8 inches in diameter? How many cubic inches does it contain?

25. How many cubic feet in a globe 2 feet in diameter?

352. Miscellaneous Examples for the Slate.

26. What will be the cost of a turkey weighing $9\frac{7}{16}$ pounds at 20¢ a pound?

27. A man owns 240 acres of Western land, which is $\frac{3}{8}$ of a township. How many acres are there in the township?

28. A shipmaster has bought $\frac{2}{7}$ of $\frac{3}{4}$ of a ship for $3000. What is the value of the ship at the same rate?

29. After $11\frac{3}{4}$ yards of a piece of linen were sold, there remained $7\frac{7}{16}$ yards. How many yards were there at first?

30. The old shilling of New England was $\frac{1}{6}$ of a dollar, and the old shilling of New York was $\frac{1}{8}$ of a dollar. How many New York shillings equalled 3 New England shillings?

31. A dealer sold 60 bunches of asparagus at the rate of 3 for 25 cents. What did he receive for them?

32. By working 10 hours a day a person can do a piece of work in $2\frac{1}{8}$ days. In how many days can he do it working $7\frac{1}{2}$ hours a day?

33. How many breadths of carpeting $\frac{3}{4}$ of a yard wide will reach across a room 18 feet wide?

34. A steamer going at the rate of 242 miles a day started from New York on Wednesday at noon. On the following Friday at noon another steamer going at the rate of 264 miles a day started on the same course. How far apart will the steamers be at the end of 10 days from Friday noon? In how many days will they be together?

35. I bought a barrel of sugar for 12¢ a pound, and I can now buy the same kind of sugar for $9\frac{1}{2}$¢ a pound. What per cent do I lose on the sugar I have left?

36. If $\frac{2}{3}$ of a barrel of beef lasts a family $3\frac{1}{7}$ weeks, how long will $\frac{5}{8}$ of a barrel last them?

37. A man put $576 in a savings bank October 10, 1877. If it gained $2\frac{1}{2}$% of interest every six months, what was due April 10, 1878?

38. A lot of paper which cost $748.60, being damaged in the store, was sold at $12\frac{1}{2}$ % less than it cost. What did it sell for?

APPENDIX.

Multiplication and Division Tables (Arts. 101, 137).

1. The pupil should be taught to write upon the slate the multiplication and division tables. Either of the following forms is shorter than that given in the book, and may on that account be preferred:

2. Multiplication of 2's and 3's (Art. 101).

First Form.

2's.			3's.		
1	2	2	1	3	3
2	2	4	2	3	6
3	2	6	3	3	9
4	2	8	4	3	12
5	2	10	5	3	15
Etc.			Etc.		

Second Form.

2's.	3's.
$2 \times 1 = 2$	$3 \times 1 = 3$
$2 \times 2 = 4$	$3 \times 2 = 6$
$2 \times 3 = 6$	$3 \times 3 = 9$
$2 \times 4 = 8$	$3 \times 4 = 12$
$2 \times 5 = 10$	$3 \times 5 = 15$
Etc.	Etc.

The reading in the first form should be, "one two is two"; "two two's are four," etc. In the second it should be, "two multiplied by one is two"; "two multiplied by two is four," etc. Or the reading in either case may be, "once two is two"; "twice two are four," etc.

3. Division by 2 and 3 (Art. 137).

First Form.

2.			3.		
2	2	1	3	3	1
2	4	2	3	6	2
2	6	3	3	9	3
2	8	4	3	12	4
2	10	5	3	15	5
Etc.			Etc.		

Second Form.

2.	3.
$2 \div 2 = 1$	$3 \div 3 = 1$
$4 \div 2 = 2$	$6 \div 3 = 2$
$6 \div 2 = 3$	$9 \div 3 = 3$
$8 \div 2 = 4$	$12 \div 3 = 4$
$10 \div 2 = 5$	$15 \div 3 = 5$
Etc.	Etc.

The reading in the first form should be, "two in two, one"; "two's in four, two," etc. In the second it should be, "two divided by two is one"; "four divided by two is two," etc.

Solutions (Art. 104, 140).

4. There are two sorts of practice, both of which the pupil needs to follow at different times during his course in Arithmetic. One consists in giving answers quickly, as soon as he hears or reads the problem, without expressing the reasoning at all; the other in giving the analysis or reasoning process according to some approved model. The former secures promptness, the latter accuracy. The teacher will best decide at what time and to what extent his class may need either sort of practice. It may be remarked, however, that set forms of reasoning should not be so rigidly insisted on as to repress originality; for oftentimes the less formal methods that come from real thinking are better than the more regular processes that come from mere imitation.

(Art. 140.)

5. The following form of solution may be preferred to the one given in Art. 140.

Solution. — Since 2 cents will buy 1 top, 1 cent will buy 1 half of a top, and 21 cents will buy 21 halves, which equal 10 tops and a half.

Ans. 10 tops and a half (or,
10 tops and half enough money to buy another top).

(Art. 117.)

6. The pupil learns by this exercise to combine adding with multiplying in a way similar to that required in multiplication where there are carryings.

The following shows the form of the exercise with 2 for the multiplier:

4	2	3	7	9	1	6	5	8
								2
9	5	7	15	19	3	13	11	17

Greatest Common Factor (page 80).

7. We have seen in Art. 191 that 6, the greatest common factor of 18 and 24, is the product of 2 and 3, the only prime

factors common to 18 and 24. *The greatest common factor of any two or more numbers is the product of all the prime factors which are common to those numbers.*

NOTE. The letters g. c. f. are used for *greatest common factor.*

8. a. What is the greatest common factor of 12, 36 and 42?

WRITTEN WORK.

$12 = 2 \times 2 \times 3$

g. c. f. $= 2 \times 3 = 6$

The prime factors of 12 are 2, 2, and 3. The product of such of these as are common to 36 and 42 must be the g. c. f. required.

We find that 2 is a factor of both 36 and 42. We find that but one 2 is a factor of 42; therefore only one 2 is used as a factor of the g. c. f. We find that 3 is a factor of both 36 and 42; therefore 3 is a factor of the g. c. f. Thus the g. c. f. sought is 2×3, equal to 6.

To find the greatest common factor of two or more numbers: *Separate one of the numbers into its prime factors, and find the product of such of them as are common to the other numbers.*

Least Common Multiple (page 80).

9. As any number contains all its prime factors, *a multiple of any number must contain all the prime factors of that number.*

A common multiple of two or more numbers must contain all the prime factors of those numbers, and

The least common multiple of two or more numbers is the least number which contains all the prime factors of those numbers.

NOTE. The letters l. c. m. are used for *least common multiple.*

10. b. What is the least common multiple of 6, 12 and 15?

WRITTEN WORK.

$6 = 2 \times 3$

$12 = 2 \times 2 \times 3$

$15 = 3 \times 5$

l. c. m. $= 2 \times 3 \times 2 \times 5 = 60$

The least multiple of 6 is 6, which may be expressed in the form 2×3.

The least multiple of 12 is 12, which may be expressed in the form $2 \times 2 \times 3$. But in 6 we have already two of the factors (2 and 3) of 12; hence, if we put

with the prime factors of 6 the remaining factor (2) of 12, we shall have $2 \times 3 \times 2$, which are all the factors necessary to produce the l. c. m. of 6 and 12.

The least multiple of 15 is 15, which may be expressed in the form 3×5. In the l. c. m. of 6 and 12 we have one of the prime factors (3) of 15; hence if we put with the prime factors of 6 and 12 the remaining factor (5) of 15, we shall have $2 \times 3 \times 2 \times 5$, which are all the prime factors necessary to produce the l. c. m. of 6, 12 and 15.

The product of these factors is 60, which is the l. c. m. sought.

NOTE. In finding the least common multiple, the factors of the given numbers seldom need to be expressed, and the written work may be greatly reduced. Thus, in this example the written work may be simply l. c. m. $= 2 \times 3 \times 2 \times 5 = 60$.

To find the least common multiple of two or more numbers: *Take the prime factors of one of the numbers; with these take such prime factors of each of the other numbers in succession as are not contained in any preceding number, and find the product of all these prime factors.*

(Art. 250.)

11. *f.* One of the following forms of solution may be preferred to that given in the body of the book:

(1.) If she had put 1 knot into a tassel, she could have made 4 tassels. If she had put $\frac{1}{3}$ of a knot into a tassel, she could have made 3 times 4, or 12 tassels; but since she put $\frac{2}{3}$ of a knot into a tassel, she could make but $\frac{1}{2}$ of 12 tassels, which is 6 tassels. *Ans.* 6 tassels.

(2.) She must have made as many tassels as $\frac{2}{3}$ is contained times in 4.

WRITTEN WORK. $\frac{\overset{2}{\cancel{4}} \times 3}{\underset{1}{\cancel{2}}} = 6.$

1 is contained in 4, 4 times; $\frac{1}{3}$ is contained in 4 three times 4, or 12 times, and $\frac{2}{3}$ is contained in 4 $\frac{1}{2}$ of 12 times, which is 6 times. *Ans.* 6 tassels.

12. From either form of solution may be derived the following general rule, which may be preferred to that in Art. **253.**

To divide by a fraction: *Multiply the dividend by the de-*

nominator of the divisor, and divide the result by the numerator; or, as more frequently stated,

Invert the divisor and proceed as in multiplication of fractions.

NOTE. This rule can be derived from the process given in Art. 250, thus:

If, in the place of 12 in the written work, we put the factors which formed 12, we shall have $(4 \times 3) \div 2$, or $4 \times \frac{3}{2}$, in which the expression for the divisor is inverted.

13. General Method of computing Interest (page 123).

The teacher is advised to have the pupils use but one method of computing interest. Some teachers may prefer the following method to that given in the book.

a. What is the interest of \$750 for 2 years 3 months 21 days at 8%?

WRITTEN WORK.

\$750
0.08
\$60.00 × 2 = \$120.00

$$\$\cancel{60} \times \frac{37}{\cancel{111}} = \$18.50$$

\$138.50

The interest of \$750 for 1 year at 8% is \$750 × 0.08, which is \$60. The interest for 2 years is 2 times \$60, or \$120. 3 months 21 days equal 111 days. The interest of \$750 for 1 year being \$60, the interest for 1 day is $\frac{1}{360}$ of \$60, and for 111 days it is $\frac{111}{360}$ of \$60, or \$18.50, which, added to \$120, makes \$138.50, the entire interest. *Ans.* \$138.50.

By this method:

1. To find the interest at any per cent for any number of years, *Multiply the principal by the rate for* 1 *year, and that product by the number of years.*

2. To find the interest for months and days, *Change the months to days, and take as many* 360*ths of a year's interest as there are days in the given time.*

NOTE. By making the divisor 365 instead of 360, *accurate interest* can be obtained.

14. Weights and Measures.

United States Money.

10 mills = 1 cent.
10 cents = 1 dime.
10 dimes or 100 cents = 1 dollar.
10 dollars = 1 eagle.

Liquid Measure.

4 gills = 1 pint.
2 pints = 1 quart.
4 quarts = 1 gallon.

Dry Measure.

2 pints = 1 quart.
8 quarts = 1 peck.
4 pecks = 1 bushel.

Avoirdupois Weight.

16 ounces = 1 pound.
2000 pounds = 1 ton.

Also sometimes used.

28 pounds = 1 quarter.
4 quarters = 1 hundredweight.
20 hundredweight = 1 ton (called long ton).

Troy Weight.

24 grains = 1 pennyweight.
20 pennyweights = 1 ounce.
12 ounces = 1 pound.

As used in mixing medicine.

20 grains = 1 scruple (℈).
3 scruples = 1 dram (ʒ).
8 drams = 1 ounce (ʒ).
12 ounces = 1 pound.

Long Measure.

12 inches = 1 foot.
3 feet = 1 yard.
5½ yards or 16½ feet = 1 rod.
320 rods or 5280 feet = 1 mile.

Square Measure.

144 square inches = 1 square foot.
9 square feet = 1 square yard.
30¼ square yards or 272¼ square feet = 1 square rod.
160 square rods = 1 acre.
640 acres = 1 square mile.

Cubic Measure.

1728 cubic inches = 1 cubic foot.
27 cubic feet = 1 cubic yard.
128 cubic feet = 1 cord.

Time.

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours = 1 day.
7 days = 1 week.
52 weeks 1 day or 365 days = 1 common year.
366 days = 1 leap-year.
100 years = 1 century.

Circular or Angular Measure.

60 seconds = 1 minute.
60 minutes = 1 degree.
360 degrees = 1 circumference.

Surveyors' Long Measure.

7.92 inches = 1 link.
100 links = 1 chain (= 4 rods).
80 chains = 1 mile.

Surveyors' Square Measure.

10000 square links = 1 square chain.
10 square chains = 1 acre.
640 acres = 1 square mile or section.
36 sections = 1 township.

Numbers.

12 units = 1 dozen.
12 dozen = 1 gross.
12 gross = 1 great gross.
20 units = 1 score.

15. Multiplication Table.

1	**1** is 1	**2** is 2	**3** is 3	**4** is 4
2	1's are 2	2's are 4	3's are 6	4's are 8
3	1's " 3	2's " 6	3's " 9	4's " 12
4	1's " 4	2's " 8	3's " 12	4's " 16
5	1's " 5	2's " 10	3's " 15	4's " 20
6	1's " 6	2's " 12	3's " 18	4's " 24
7	1's " 7	2's " 14	3's " 21	4's " 28
8	1's " 8	2's " 16	3's " 24	4's " 32
9	1's " 9	2's " 18	3's " 27	4's " 36
10	1's " 10	2's " 20	3's " 30	4's " 40
11	1's " 11	2's " 22	3's " 33	4's " 44
12	1's " 12	2's " 24	3's " 36	4's " 48

1	**5** is 5	**6** is 6	**7** is 7	**8** is 8
2	5's are 10	6's are 12	7's are 14	8's are 16
3	5's " 15	6's " 18	7's " 21	8's " 24
4	5's " 20	6's " 24	7's " 28	8's " 32
5	5's " 25	6's " 30	7's " 35	8's " 40
6	5's " 30	6's " 36	7's " 42	8's " 48
7	5's " 35	6's " 42	7's " 49	8's " 56
8	5's " 40	6's " 48	7's " 56	8's " 64
9	5's " 45	6's " 54	7's " 63	8's " 72
10	5's " 50	6's " 60	7's " 70	8's " 80
11	5's " 55	6's " 66	7's " 77	8's " 88
12	5's " 60	6's " 72	7's " 84	8's " 96

1	**9** is 9	**10** is 10	**11** is 11	**12** is 12
2	9's are 18	10's are 20	11's are 22	12's are 24
3	9's " 27	10's " 30	11's " 33	12's " 36
4	9's " 36	10's " 40	11's " 44	12's " 48
5	9's " 45	10's " 50	11's " 55	12's " 60
6	9's " 54	10's " 60	11's " 66	12's " 72
7	9's " 63	10's " 70	11's " 77	12's " 84
8	9's " 72	10's " 80	11's " 88	12's " 96
9	9's " 81	10's " 90	11's " 99	12's "108
10	9's " 90	10's "100	11's "110	12's "120
11	9's " 99	10's "110	11's "121	12's "132
12	9's "108	10's "120	11's "132	12's "144

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